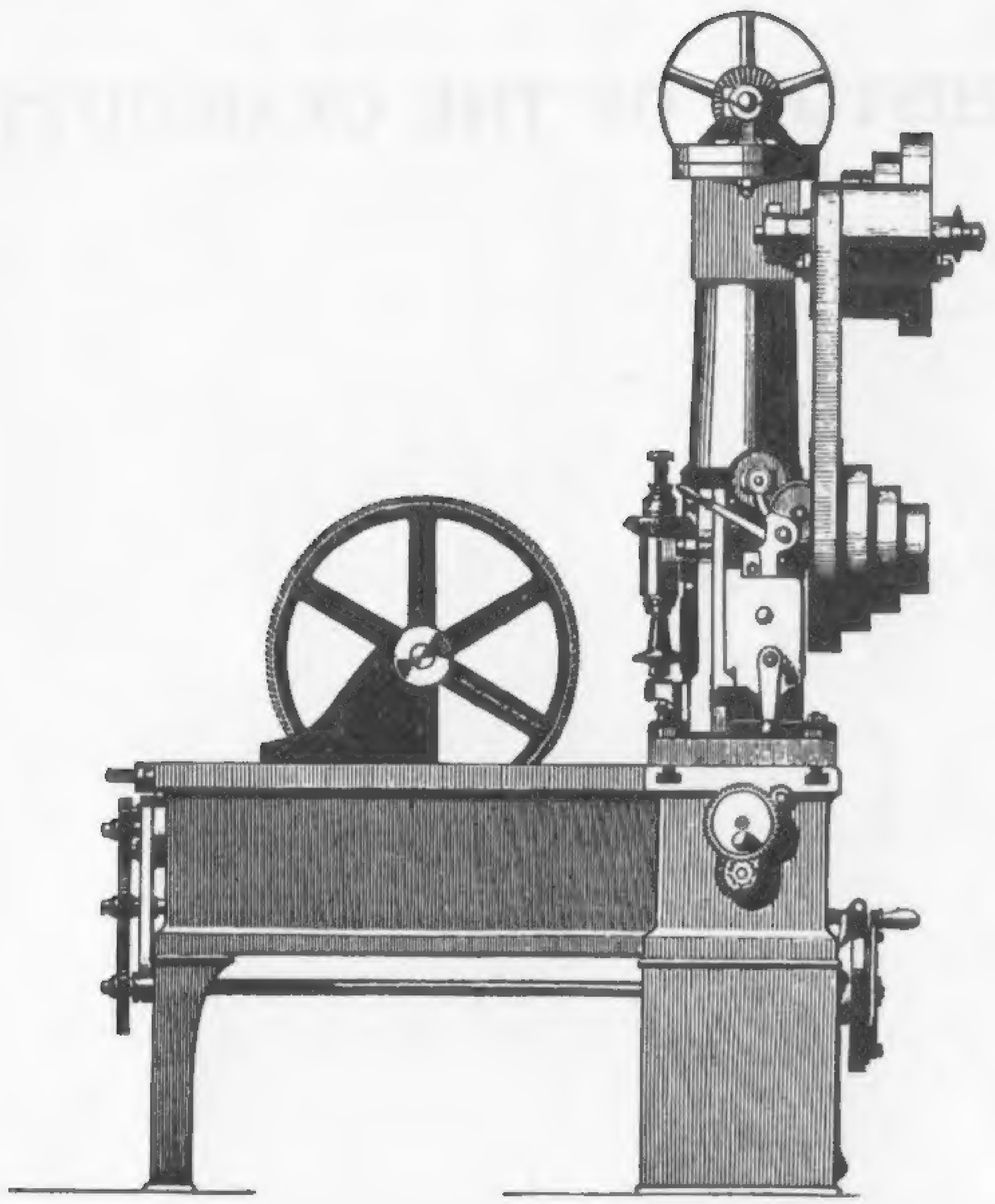

Robert S. Woodbury

HISTORY OF THE GEAR-CUTTING MACHINE

A Historical Study in Geometry and Machines

with a Foreword by Abbott Payson Usher

For

C. HENRY BLETZER

Master Toolmaker

My Kindly Teacher

and

My Friend

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FOREWORD

This stimulating book demonstrates vividly the importance of monographic studies in the history of technology. General histories and biographical studies of engineers and inventors contribute much breadth of vision and many essential details, but the long sequences of events in particular fields are obscured by the diffusion of attention over large areas, or by concentration on short periods of time. The history of gear-cutting mechanisms begins early in the sixteenth century, and may even be associated with some of Leonardo da Vinci's drawings. These mechanisms developed actively until 1930, when the primary types seem to have become stabilized. The entire sequence, thus, covers four centuries. It was a period of intense activity in all fields of science and technology, so that these aspects of the historical development would be competing in every generation with powerful interests created by events in other fields. Monographs are, therefore, of great importance, and most particularly in a field that may seem to be of secondary interest. Even for the nineteenth century, the literature on machine tools has not been abundant, and the gear cutters have attracted less attention than other tool-making machines. The present monograph is, thus, a notable addition to the literature.

The history of gear-cutting mechanisms brings into special prominence the convergence of sequences of events that are, at the outset, very loosely associated, or even entirely independent. This phenomenon of convergent synthesis is characteristic of the history of invention in all fields, but it is not always conspicuous. Many writers, too, ignore the phenomenon. They present the history of science or technology as a movement from the implicit to the explicit, so that they describe the sequences of events as closely organized linear sequences stemming from a single center of implication. It is, thus, of great significance to see the phenomenon of convergence appear as vividly as it does in the present monograph.

In the sixteenth century, we find two clearly distinguished points of initiation. There is an approach through the mathematical analysis of curves and the geometry of motion. There is, also, an empirical approach through the efforts of clock makers to produce machines to cut gears for their wheel work. Both lines of development proceeded with little direct contact until the close of the eighteenth century, an unusually long period of slow convergence. In each sequence many steps can be distinguished. The cycloidal curve was first studied by Nicholas of Cusa in 1451. The epicycloids were discovered by Albrecht Dürer in 1525. Jerome Cardan studied the relation of geometrical curves to gears, but without full mathematical analysis (1557). Full analysis was notably advanced by the scientists of the first half of the seventeenth century. Gears with epicycloidal teeth were constructed in mid-seventeenth century by Desargues and Rømer, though the dates of their achievements are uncertain. Extended mathematical analysis of gear teeth begins effectively with Phillipe de La Hire, in 1694. In addition to his treatise, he designed gearing for a large water works. The treatise discussed the whole family of epicycloids, and reached the conclusion that the involute curve was the best of all the exterior epicycloids. The gap between theory and practice is vividly shown by the fact that the involute form for gear teeth was not adopted in practice for 150 years. Mathematical analysis was further advanced by Euler (1754-55), but the involute curve was not mentioned by Camus (1733, 1766) in a treatise that had wide currency. Kaestner, writing in 1781, thought of his work as a systematic treatment of a stable and complete body of knowledge. This treatise may, therefore, be accepted as the close of the work on the abstract analysis of the geometry of motion in wheels and gears.

When we consider the limitations of the primary machine tools in the sixteenth and seventeenth centuries, it is remarkable that we find such a substantial series of efforts to produce mechanisms capable of cutting gears for clocks and watches. Practical accomplishment was possible, because the wheel work of clocks did not require great precision or refinement of design. The wheels were counting devices that used little power. The precision mechanism was the escapement, and until the pendulum was introduced the escapement was a simple device that displayed little refinement in execution. Much attention was given to the escapement in the eighteenth century, but the problems were distinctive.

There are drawings in Leonardo da Vinci's Note Books that suggest a mechanism for cutting gears; but there is little text description, and we have no certainty that any machine was built. In 1540, however, we are told that Juanelo Torriano constructed a machine to cut teeth in the wheels required by the planetarium, or astronomical clock, built for Charles V. It was necessary to make 1800 wheels. These were cut at the rate of three each day, over a period of three years and a half. It is presumed that a rotary file was used. This machine came into common use in Spain.

About 1670 Robert Hooke invented a wheel-cutting machine, which was later described by Le Roy, the French clock maker. The Science Museum, at Kensington, has a wheel-cutting machine, dated 1672, that is similar in general design to Hooke's invention. The primary features of the wheel-cutting engines of the eighteenth century are found in this machine of 1672, but there is development of many additional features. The cutting tools assume new shapes in the machines of Biot (1702), and Leupold (1724). In the decade of the thirties, Polhem produced an automatic machine. The devices for automatic control had no influence on other machine designers, but the general conception of Polhem's engine was clearly influential in France. Important work was also done in Great Britain by Rehé and Hindley. Use was made of mathematical analysis; especially the treatise of Camus, but the clock makers' engines fell short of becoming instruments of precision. Their construction, too, was not heavy enough to carry them beyond light engineering.

The convergence of theory and practice came, in this field, after the development of heavy engineering work for the application of steam power. Castings and hand-cut wheels were used in the early years, but the extension of transmission systems in the field of prime movers called for

major improvements in gear-cutting engines. The theoretical work of the eighteenth century was adapted to the needs of engineers by a group of writers, happily called the translators. The outstanding figures were Hawkins (1806) and Willis (1841). Buchanan's manuals of 1806 and 1833 simplified the treatises of Camus and de La Hire sufficiently to make them available to competent shop men, without engineering training. Heavy duty gear-cutting machines began to appear as early as 1800. In the course of fifty years, a variety of types were developed that met the requirements of the machine-tool industry of that time. These achievements and the important treatise of Sang (1852) mark the bridging of the gap between theory and practice. The rapid growth of the engineering trades, however, created new needs; and vast effort was put into the refinement and perfection of the gear-cutting engines as precision instruments. Between 1867 and 1930, some 2,344 patents were taken out in the United States for mechanisms for cutting gear teeth. Between 1849 and 1895, there were 100 patents granted for gear cutters proper.

The history of gear cutting, thus, demonstrates the need of revision of both of the more common concepts of the history of science and technology. The romantic concept of invention is clearly inadequate, because there are many more steps in the process than is presumed by those who think in terms of rare acts of inspiration. The concept of a simple linear development is also unsound. Inventions do not emerge directly and inevitably from specific generalizations in the pure sciences, nor from a practical achievement in producing or controlling specific modes of motion. The records show so much interdependence, that the development can be adequately described only as a form of multilinear process.

ABBOTT PAYSON USHER

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PREFACE

Although the historians have treated the political, economic, and social aspects of the Industrial Revolution very fully, only a few have looked at the technological side. Even these have been concerned only with *power* (principally the steam engine), new *materials* (mostly steel), and the many types of *production machinery* (principally for textiles). None have considered that technical development without which the steam engine and the machinery could not have been built, the development without which steel would have been of little significance—the *machine tool*. One can hardly say that the machine tool was a sufficient condition for the Industrial Revolution, but we may be certain that it was a necessary condition for the development of the industrial society in which we live.

While in this study we shall point out the broader implications of machine tools which are clearly indicated by careful study of the sources, it is essential to avoid the premature interpretations so common in studies of the development of power, steel, and the textile industry—premature because made too glibly before the facts of the actual hardware have been established. This history, then, will be characterized by two principles of approach: (1) careful examination of primary sources, such as patents, catalogues, trade journals, actual machines; and (2) careful consideration of the technical development—without which there would have been no economic or social effects.

This monograph is one of a series planned to provide the scholarly foundation essential for writing a History of Tools. Several of the monographs have been completed. The program is planned to result in publication of a two-volume *History of Metal-Cutting Machine Tools* in late 1961. Other volumes on Hand Tools, Woodworking Tools, and Metal-Forming Tools will follow later.

In this monograph on the History of the Gear-Cutting Machine no attempt has been made to do what is probably even more important, to write a general history of gears and gearing. In the past the theory of gears has dealt only with types of individual gears. A beginning has been made on the theory of linkages, but de Latil's application of his theory of artificial effectors to the logic of mechanisms shows the limitations of our present theory of gears. Gear-cutting machines and the theory of the gear itself are important elements of the larger problem, for, as in the electronic type of servomechanisms, the gearing types of mechanical servomechanisms were limited to what could be manufactured with existing machines. Until the automobile there was little demand for new types of gears or gear trains; the machine designer simply accepted the limitations of the types available to him in various "handbooks of ingenious mechanisms." The new science of cybernetics has opened up great possibilities here, certainly for historical analysis, perhaps for new designs. But we shall attempt only the lesser task of description and analysis of the development of the machines on which gears have been and still are made.

My gratitude for scholarly assistance on special points is indicated in the text. But two others deserve special credit—Cynthia Cooley and Margaret Little, of the Reference Department of the M.I.T. Library, whose perseverance and untiring patience with my unavoidably imperfect bibliographical data located many books and references both rare and difficult to find.

ROBERT S. WOODBURY

Cambridge, Massachusetts
October 1, 1957

INTRODUCTION

In the technical sense, nearly all gear-cutting machines have been only milling machines, shapers, or slotters adapted to a specialized purpose.[1] Yet the gear-cutting machine is of particular interest in the history of tools. Probably none of the machine tools has called for more knowledge of geometry and more ingenuity of design to solve its unique problems. Although it is a specialized machine, it has appeared in types utilizing many different ways of doing the same job. It stands, therefore, in marked contrast to the milling machine and other tools, which as they became of more general application, tended to have standardized forms. The gear-cutting machine has become more and more specialized, and at the same time has evolved an amazing variety in design and construction.[2]

The gear-cutting machine is of special interest, too, because it is designed to solve *in metal* some very complex geometrical problems with a high order of accuracy. And, curiously enough, it had considerable success in the production of gears long before the geometrical problems were completely solved even theoretically. From the beginning of gear cutting there was a strong tendency toward precision in metallic gears.[3] By 1700 the attention of geometers was attracted to the theory of the form of gear teeth; not, however, by the precision needs of clock or instrument makers, but by the engineers' practical interest in mills and water power. The metallic gear by its very nature must be precise if it is to run smoothly and quietly under substantial loads and at high speeds.

1. Exceptions are the few machines using the impression method. See page 113 below.
2. For photographs, drawings, and descriptions of nearly every type in use in America and in Europe as of 1909, see Ralph E. Flanders, *Gear Cutting Machinery*, N. Y., 1909, *passim*.
3. See Morales, p. 45 below.

However, the practical men had been able by various empirical means to get gears adequate for their needs, at least until the early 19th century, when the mathematicians' work was translated into practical language.

One is led to wonder why the makers of precise devices such as clocks and mathematical and astronomical instruments were not more concerned with precision teeth for their gears. Indexing—division of the circumference of a circle into a desired number of equal parts—had already become a problem for both these craftsmen. The instrument makers were devoting a great deal of care to the precise division of a circle for the exact measurement of angles in surveying and in astronomy. Allied with this problem was that of making a precision screw. In fact, the two problems[4] are theoretically identical and come to have a practical significance for us here in the case of worm and spiral gearing.

The exact division of the circle had already entered into the design of gear-cutting machines, as the clock makers had discovered in making clock wheels. Because even a pretty poor gear must come out to an exact whole tooth, the clock makers of the mid-18th had already met the problem of accurate indexing.

The acceptance by these two groups of precision craftsmen of purely empirical solutions for the form of their gear teeth can only be accounted for by the fact that in both cases their gears operated at low speeds and under small loads, and in the case of clocks, since they ran in only one direction, backlash did not have to be considered.

The interest of the mathematicians, such as Desargues, de La Hire, Euler and Camus, seems to have arisen from a desire to increase efficiency and reduce wear in mills of various types, where, although the speeds were low, the load was substantial.

4. Both will be considered in detail in a later paper on the History of Shop Precision of Measurement.

Questions of exact tooth form, pressure angle, and strength did not enter into the designs of the clock and instrument makers. And since they did not have to provide for interchangeable sets of gears, involute teeth were not required. All these questions were to arise in the design of *production* machinery after 1800.[5]

The early attainment of precision in metallic gearing led to an economic importance for gear cutting that might at first not be suspected. Machinery of the early industrial revolution was largely belt-and-pulley driven, and such gears as it had were very crude. This meant that these machines had to be operated at relatively low speeds and that internal coordination of the operating parts was always hampered by belt slippage and by gear irregularities. To avoid these troubles the machine designer of that day often used linkages where we would use gears. It is no coincidence that the higher speeds and greater complexity of machinery in general appear as soon as the gear-cutting machine becomes a production tool. This can be traced very easily in textile machinery. The availability of accurate gearing in quantity also made possible the sewing machine, the rotary printing press, the automobile,[6] and many other familiar machines.[7]

5. Smeaton had introduced the iron wheel at Carron in 1754, but these were rough castings only; no attempt was made to form the teeth accurately. Murdock, Watt's assistant, had used similar gears in a mill a little later, but their work had little influence generally and was no real mechanical improvement.The transition really comes with Rennie's Albion Mill, which he built between 1784 and 1788. Here he made the shafts and entire wheels of wrought and cast iron. These gears had their teeth carefully chipped and filed into epicycloids. (See Samuel Smiles, *Lives of the Engineers*, London, 1861, Vol. II, pp. 138–139.) After Rennie iron gears were soon adopted generally in all large machinery.

6. Henry J. Eberhardt, *Influence of the Automobile on Gear Cutting and Gear Cutting Machinery*, Meeting Am. Soc. Mech. Eng., May 26–27, 1921, partially reprinted in *Mechanical Engineering*, Aug. 1921. Luther D. Burlingame, in *Motor World*, Mar. 10, 1915.

7. For the significance of gearing as a mechanical servomechanism see Pierre de Latil, *Thinking by Machine*, Boston, 1957, pp. 84-86.

The gear-cutting machine will also prove to be of interest because the work it does is complex, and therefore the required skill had to be built into the machine from the beginning. However, once set up, its operation is routine and quite simple. This fact, taken together with the enormous demand for already fairly well standarized gears, led to the gear-cutting machine becoming fully automatic relatively early, in fact at about the same time as another machine with these same characteristics, the screw machine.[8]

The gear-cutting machine, the milling machine, and the turret lathe belong to the second generation of machine tools, after the classical development in Maudslay's day of the screw-cutting lathe, the planer and the shaper, for gear cutters were common only after about 1850, and generating type machines only after 1875.[9]

The history of a machine tool which at first glance seems of interest only to the specialist need, then, make no apology to the mechanic, the engineer, the geometer, or to the economic historian.[10]

8. This development will be taken up in a later paper on the History of the Turret Lathe, where the application of de Latils' Theory of Effectors to this machine leads to some interesting results, especially in evaluating the work of Spencer.

9. Howard A. Coombs, in *American Machinist*, 1903, p. 1013.

10. The work of Conrad Matschoss, *Geschichte des Zahnrades*, Berlin, 1940; Karl Kutzbach, *Bemerkungen zur Entwicklung der Verzahnung*, Berlin, 1940, and his papers in *Zeitschrift des Vereins Deutscher Ingenieure*, 1924, pp. 913, 1075, and 1105, and 1927, p. 73; Franz M. Feldhaus, *Die Geschichtliche Entwicklung des Zahnrades in Theorie und Praxis*, Berlin, 1911; and O. Kammerer, "Die Entwicklung der Zahnräder" in *Beiträge zur Geschichte der Technik und Industrie*, Bd. IV., 1912, pp. 242-273, while of great use to the author in providing clues, are incomplete and too frequently require re-examination of the sources. It has therefore seemed worthwhile to provide in English a history of the gear-cutting machine and of the mathematics of gears, based upon careful examination of all the sources.

I The Mathematics of Gears

THE GEOMETERS

THE TRANSLATORS

THE SCIENTIFIC MECHANICS

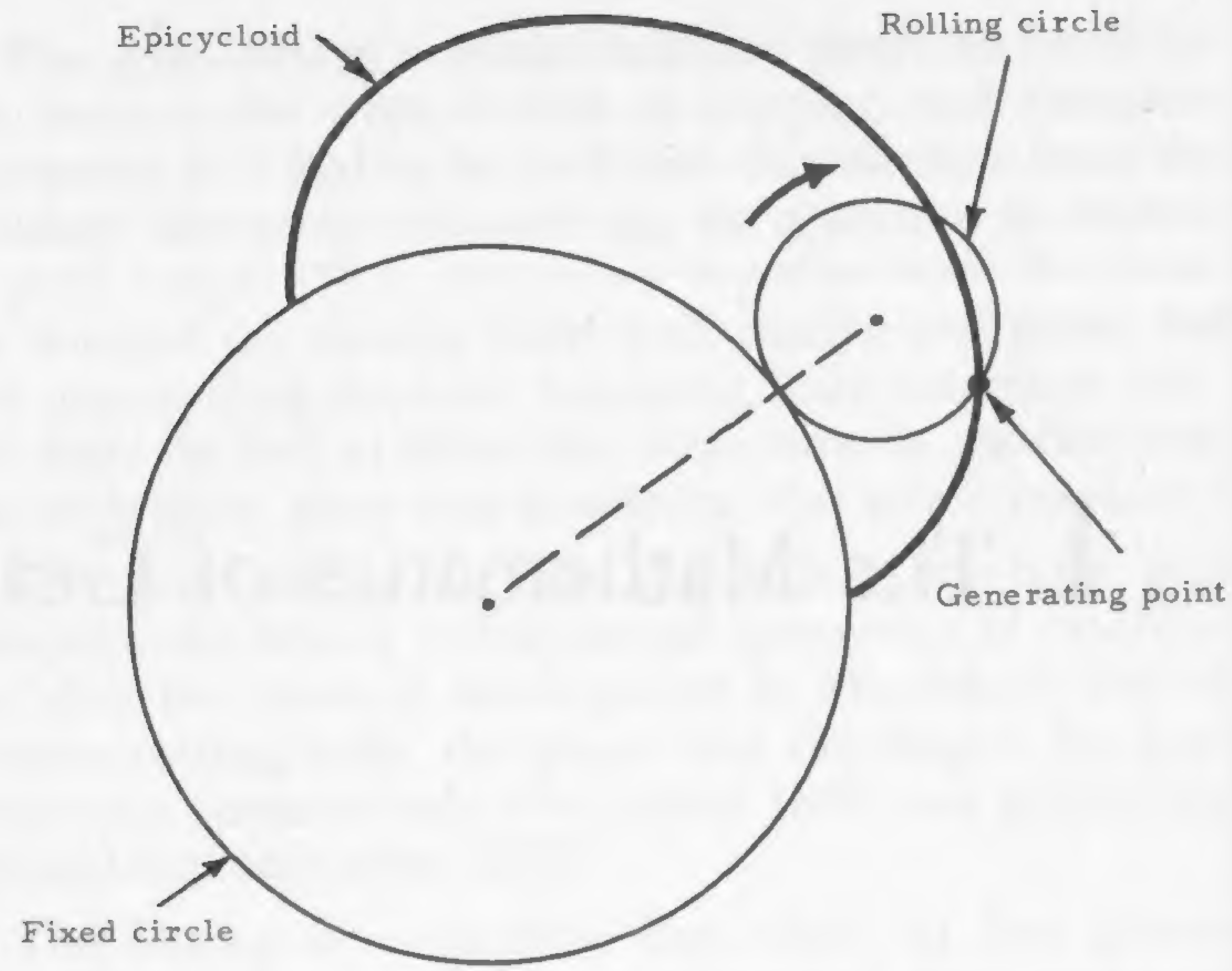


FIG. 1A. THE EPICYCLOIDAL CURVE

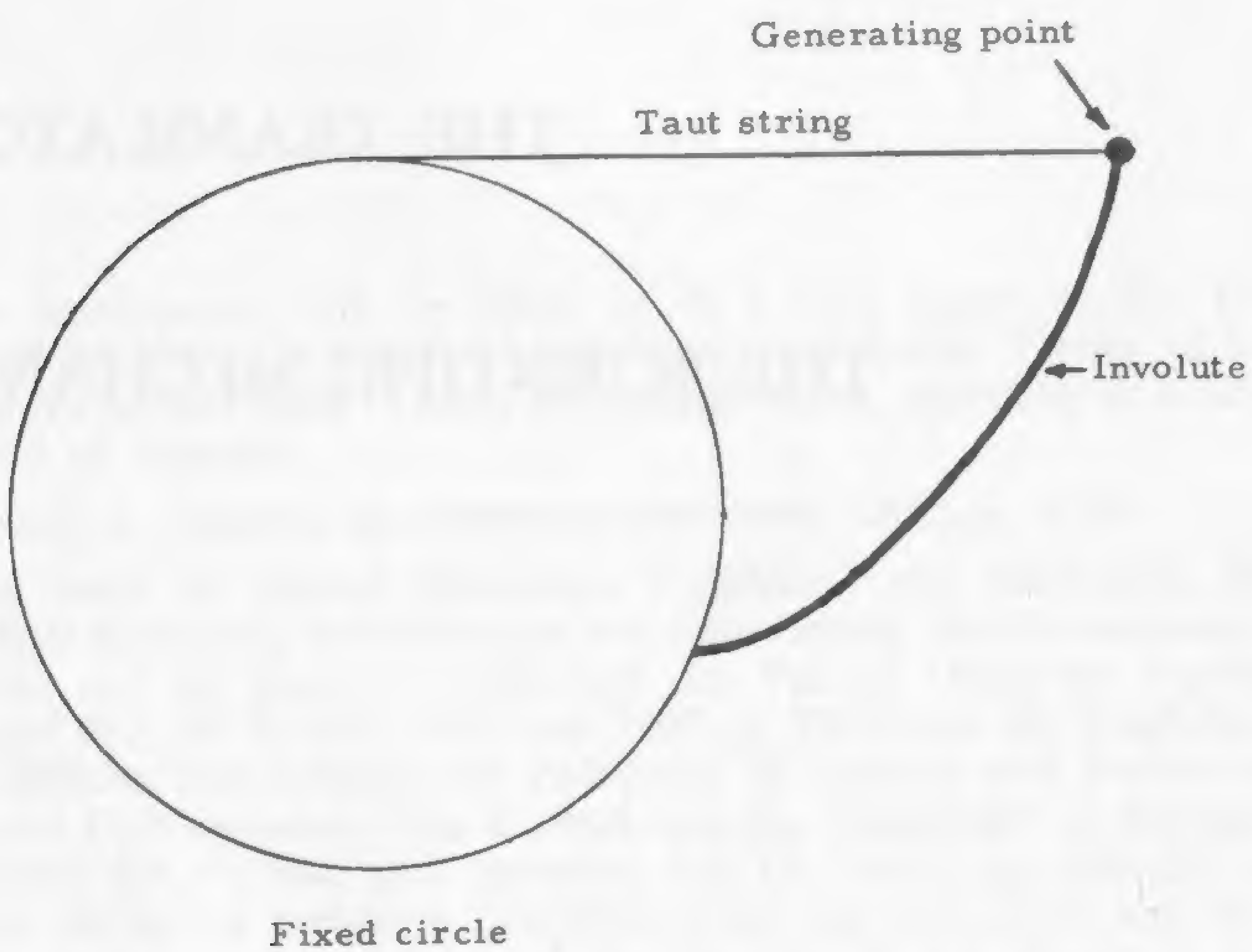


FIG. 1B. THE INVOLUTE CURVE

The Geometers

The historical relationship between gear geometry and gear-cutting machines is a curious one, in that both made considerable progress on their common problem for nearly a hundred years before either was much aware of the other's existence, much less of what they could do for each other. In the eighteenth century only a few scientists, such as Réaumur, and even fewer mathematicians,[1] were interested in the problems of the engineer, to say nothing of those of the mechanic. To be sure, in the 17th and 18th centuries there were close relationships between the scientists and the instrument and clock makers, but their mutual interests seem not to have extended to the scientific study of gears. The "mechanicians", as they were called, were content with empirical solutions.

The mathematicians' interest was aroused somewhat earlier. The basic problem in the scientific design of gears until the late 19th century was to determine the curve for the profiles of their teeth to give continuous contact with the minimum of friction. Although a number of curves theoretically meet these conditions, practice has centered around two members of the cycloid family—the epicycloid and the involute. The epicycloid is the curve generated by a point on the circumference of a circle as it rolls on the outside of a fixed circle. (Fig. 1a) The involute is the special case of the epicycloid generated when the radius of the fixed circle is infinite and therefore becomes a straight line. (Fig. 1b). There are, however, certain practical considerations which led to a long controversy over which of these two curves was to be preferred. The cycloidal curve had

1. *E.g.,* Euler's studies on the windmill. "De constructione Aptissima Molarum Alatarum," in *Acad. Sci. Imp. Petro., Novi Commentarii,* vol. IV, 1743, pp. 41-108. In the seventeenth century Desargues is an exception.

first been studied by Nicholas of Cusa (1401-1464) in 1451.[2] The epicycloid had been discovered by Albrecht Dürer (1471-1525) in 1525.[3] The first, empirical, mathematics of gears in print is in Cardano in 1557.[4] In the 17th century Galileo, Torricelli, Descartes, Roberval, and Mersenne had studied the properties of this family of curves. In early 1658 Pascal created quite a controversy with his Dettonville Problems,[5] in which Christopher Wren, Wallis, and Lalouvère had been interested. By the middle of the 17th century the mathematics of the cycloids had been very well worked out from purely mathematical interests.

Leonardo da Vinci had shown some drawings of gear tooth form—one very like a buttress tooth and another looking much like modern teeth, but he does not say enough in the text for us to be sure.[6] The French mathematician Gerard Desargues (1593-1661) also had interests in architecture and engineering. In the course of building some machinery near Paris he designed and constructed the first gears having epicycloidal teeth,[7] probably in 1644-1649 but

2. Nicolaus Cusanus, *Opera,* Paris, 1514, Vol. II, 33-59.

3. Albrecht Dürer, *Underweysung der messung mit dem Zirckel und richtscheyt,* n. p., 1525, pp. 6-17.

4. Girolamo Cardano, *De rerum varietate,* Basel, 1557, pp. 263-372.

5. For Pascal's side of the story see: Blaise Pascal, *Oeuvres,* La Haye, 1779, t. V, pp. 135-275.

6. Theodor Beck, *Beiträge zur Geschichte des Machinenbaues,* Berlin, 1899, p. 100 and fig. 100. It is clear that Leonardo was interested in reducing the friction of gear teeth. See his roller tooth gears in Codex Atlanticus, fol. 391.

7. See letter of Christain Huygens to Lodewijk Huygens, 29 Oct. 1671, in C. Huygens, *Oeuvres de Christiaan Huygens,* La Haye, 1888-1950, Vol. III, pp. 112-113. Also see Phillipe de La Hire, "Traité des épicycloides et leur usages dans la mécanique", *Mém. de Math. & Phys. de l'Acad. Roy. Sciences,* Paris, 1694, t. IX, p. 342; also published separately as La Hire, *Mémoires de Math. et de Phys.,* Paris, 1694, Préface, where de La Hire says he worked on the problem twenty years before. This would be about the same time as Rømer. And La Hire, *Traité de Méchanique,* Paris, 1695, Préface, p. 10, pp. 368-374. It is not mentioned in Desargues' published works, nor is it among his papers; see René Taton, *L'Oeuvre mathématique de G. Desargues,* Paris, 1951, p. 65.

possibly in 1657-1661. However, both Leibniz and Wolff[8] say that this was first done in 1674 by Ole Rømer (1644-1710), the Danish astronomer who first measured the speed of light. This discovery is not to be found in Rømer's published works, but since his papers were unfortunately lost in a fire in 1728 at Copenhagen we have no choice but to accept Leibniz' and Wolff's statements as evidence of an independent discovery.

The work of Desargues and Rømer, however suggestive, cannot compare to that of Phillipe de La Hire (1640-1718), who in 1694 made the first systematic application of the epicycloid to gear teeth.[9] He is also said to have applied his discoveries to the design of a large waterworks. Although his claim to be the first to apply the cycloids to gear tooth forms cannot be sustained, he deserves far greater credit as the first to treat gear teeth mathematically and systematically. It was he who first laid down the basic geometrical principles

8. Gottfried Wilhelm Freiherr von Leibniz, *Societati Regiae Scientiarum, Miscellanea Berolinensia,* Berlin, 1710, Vol. I, "Tentamen natura et remediis resistentiarum. . .," p. 315. Leibniz does not give any date; he only says that it was done while Rømer was at the Royal Observatory in Paris. See also Leibniz' letter to Wolff of 20 August 1705 in C. I. Gerhardt (ed.), *Briefwechsel zwischen Leibniz und Christian Wolff,* Halle, 1860. But see *Vivorum celeberr. C. G. Leibnitii et Johan. Bernoullii Commercium philos. et math.,* Lausannae et Genevae, 1745, Vol. I. pp. 347, 349 and 352, where Bernoulli credits Desargues. Christiano L. B. de Wolff, *Elementa Matheseos Universae,* Halle, 1758, t. II. p. 302, par. 960. (1st ed. 1713)

See also Joanne-Baptista du Hamel, *Regiae Scientiarum Academiae Historia,* Lipsiae, 1700, p. 162, par. 1 anno 1675; and Christiaan Huygens, *Oeuvres complètes,* La Haye, 1934, t. XVIII, p. 607-620, where Huygens credits the discovery repeatedly to Rømer in work of his own on the same subject written in late 1674. The editors of Huygens seem to feel that de La Hire should also be given credit for an independent discovery, but they ignore Desargues' claim entirely. They state (t. XVIII, p. 602) that Rømer's paper to the Academy has not been preserved.

For much assistance on this problem the author must acknowledge his indebtedness to the *Danmarks tekniske Bibliotek* and to Dr. Palle Birkelund, Director of *Det Kgl. Bibliotek* of Copenhagen, who kindly supplied several references, as well as microfilm of Huygens.

9. Phillipe de La Hire, *Traité des Epicycloides,* Paris, 1694, and his paper in *Mém. Acad. Roy. Sci.* cited above.

of gear design: (1) the aim of securing uniform pressure and uniform motion,[10] (2) tooth surfaces designed to roll on each other and so avoid all friction,[11] and (3) the principle that if a tooth of a gear is formed by a part of an exterior epicycloid described by *any* generating circle, the tooth of the follower will be a portion of an interior epicycloid described by the same generating circle. For a given tooth form he shows how to find the corresponding tooth form that will work with it. To do this he uses the principle of uniform force and motion to combine the given tooth form with an epicycloid. De La Hire does this for several given tooth forms, but points out that although in theory it can be done for *any* tooth form, in practice some are impossible.[12]

De La Hire considered the involute as the best of the exterior cycloids, since he recognized that it is the special case where the generating circle's radius is infinite. He also noted that the involute tooth gives the teeth of the corresponding rack as having straight sides.[13] But it was to be 150 years before this principle found practical application.

The invention of the bevel gear has often been credited to de La Hire, as well as the correct recognition of the principle upon which geometrical analysis of the bevel gear is based. Neither of these is the case. He had shown the conical trundle as a means of changing the direction of transmission of motion,[14] but this had been known long before.[15]

10. This is given more explicitly in his *Traité de Méchanique,* Paris, 1695, p. 213. Partial English translation by Venterus Mandey, and James Moxon, *Mechanical Powers,* London, 1690. This principle was to some degree anticipated by Huygens, t. XVIII, p. 607, in 1674. "equali vis continue in se mutuo agentes."

11. In the latter part of this requirement de La Hire was mistaken, as Camus was to show. But the aim still persists, and many practical mechanics still believe that all gears do this.

12. de La Hire, *Traité de Méch.,* Book X, prop. III and VI.

13. de La Hire. *Traité de Méch.,* prop. LXVI.

14. de La Hire, *Traité de Méch.,* pp. 217-218

15. Jacques Besson, *Theatrum instrumentorum et machinarum,* Lugdini, 1578, shows a bevel gear, crude and only at right angles, in Fig. 27. A. Ramelli, *Le Diverse et Artificiose Machine,* Paris, 1588, Para. 1580, ch. XLVIII, shows one more highly developed and suited for any convenient

Nor is his analysis of the trundle any basis for that of bevel gears—for, as Hawkins points out,[16] the cones are in opposite directions.

Long before Hawkins and Willis described the many advantages of the involute tooth, the mathematics of the involute curve and its application to gear teeth had been worked out by Leonhard Euler (1707-1783), the great Swiss mathematician. In his first paper Euler already shows the grasp and precision of his great mathematical mind. He specifically states the conditions as:

1. Uniform rotary motion of both gears.
2. In the mutual action of the teeth *nullus atritus oriatur.*

He has the principle of the common tangent. Euler specifically points out the need for proper design of gear teeth to avoid friction and wear and indicates this application to clocks. Most clock makers, however, ignored this, if they ever heard of it. Euler's treatment of gear teeth was very general and carried out by the application of principles of analytic geometry using both differential and integral calculus. He sets up in Section 1 the mathematical expressions for gears to move without friction of their teeth (actually for a minimum). In Section 2 he sets up the expressions for gears to move with uniform motion. Then he shows that the equations developed in Sections 1 and 2 can be satisfied only by involute or epicycloidal teeth.[17]

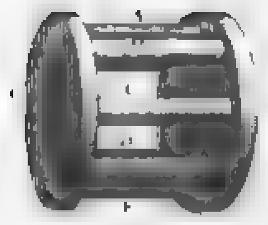
In a later paper[18] Euler shows how to construct the figure of the teeth, but only in theory, although he does give both an approximate and a precise method. He also shows

angle. Still earlier a highly developed *pair* of bevel gears (but acting only at a right angle) is shown in Leonardo da Vinci, *Codice sul Volo degli Uccelli e Varie Altre Materia* (written at Florence in 1505), Milan, 1946, fol. 9. Also in Codex Atlanticus, fol. 397Rb.

16. See below, p. 22.

17. Leonhard Euler, "De Aptissima Figura Rotarum Dentibus Tribuenda," in *Academiae Scientiarum Imperiales Petropolitanae, Novi Commentarii,* 1754-55, t. V, pp. 299-316. The solution is p. 315, para. 22.

18. *Idem.* "De Figura Dentium Rotarum," 1765, t. XI, pp. 207-231.



how to determine the "amplitude" of the mutual action of the teeth and in so doing assumes a pressure angle of 30°. Matschoss[19] says that Euler put the theory of teeth in a form which seventy years later the machines could use and is therefore the "father of involute gearing." The fact is that the men who designed the machines for generating involute teeth come nearly one hundred years after Euler and never used his work. Its mathematics was far beyond the capacity of the practical men who actually designed the gear-cutting machines. If Euler be the "father of involute gear teeth" later insemination was required by Hawkins and Sang and even then a confinement of some thirty years was required before Beale delivered the child at Brown and Sharpe!

The first mathematician to work the theory of gear teeth into a systematic and general theory of mechanism was Charles Étienne Louis Camus (1699-1768). However, since he often introduced the notion of *force*, his theory is not that of pure mechanism.[20]

Camus repeats much of de La Hire's work, but adds many important elements of his own. He gives a detailed analysis of the teeth desirable for the combination of a spur and a lantern gear. Here we see clearly the influence of wooden mill gearing.[21] He even considers the case of the crown gear and the beveled lantern.[22] In Camus we can also see some influence from the clock makers, for in his figures 169, 171-177, and elsewhere the teeth of his pinions often have the "bay-leaf" form and are shown with considerable backlash. (Fig. 2)

Camus does, however, correct de La Hire in that he recognizes the fact of sliding of even the epicycloidal teeth

19. Conrad Matschoss, *Geschichte des Zahnrades,* Berlin, 1940, p. 68.

20. Charles Étienne Louis Camus, "Sur la figure des dents des roues et des ailes des pignons", in *Histoires et Mémoires de l'Academie des Sciences,* Paris, 1733 p. 165, and later included in his *Cours de mathématique,* Paris, 1766. Books X and XI were translated as *Teeth of Wheels* by John Issac Hawkins, London, 1806.

21. This had been treated more empirically by Jacob Leupold, *Theatrum machinarum generale,* Leipzig, 1724, p. 49 and Fig. XIV.

22. Camus, *C. Math.* Figs. 200 and 201.

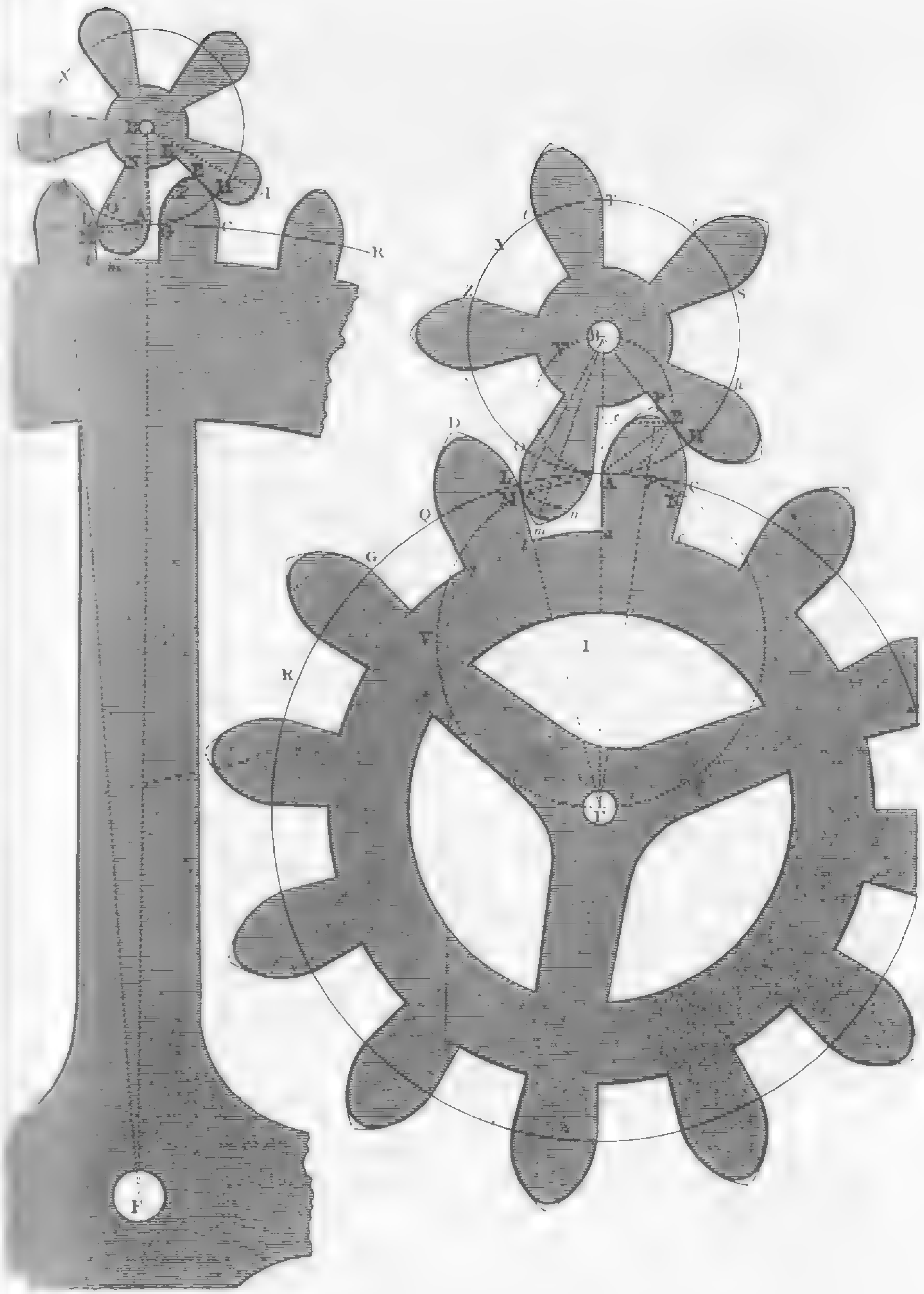


FIG. 2. CAMUS' EPICYCLOIDAL TEETH, 1733 (*Hawkins*)

one upon the other and that this phenomenon is one of the principal sources of friction and wear in gearing.

The action of engaged teeth relative to the line of centers is discussed, and he points out that the action is best when engagement takes place after the working face of the driving tooth has passed the line of centers—receding action.

Camus goes on to consider the problem of the minimum number of teeth and that of the proper form for the ends of the teeth. He deals with true bevel gears and uses the rolling-cone principle for their analysis. But he considers only the case of a crown and a bevel gear interacting. However, the more general case would require only a more specific enunciation of the principle of his figure 199.

Camus does not consider the involute tooth at all. Although he analyzes trains of gears in Book XI, he says nothing of the form of teeth required in a series of three or more gears. This can probably be accounted for by the fact that he seems to have only clockwork in mind. The mills of this era seldom had trains of more than two gears engaged.

Clearly Camus had the basis for a theory of mechanism of gear teeth, but it was not systematically and completely worked out, as in Willis.

In 1781 Abraham Gotthelf Kaestner (1719-1800) took up the problem.[23] He is thoroughly familiar with the writings on gears of Leibniz, de La Hire, Camus, Euler, and especially Bernoulli's principle of the use of the normal to the curves in their analysis.[24] Kaestner modestly disclaims having anything really new, only that he has put it in more useful form. This in itself was new and important, but he had significant contributions of his own to make.

To be sure he does show a simple method of computing the teeth of both epicycloidal and involute forms. *This was the first step in making the work of the geometers available to the practical men.*

23. Abraham Gotthelf Kaestner, "De Dentibus Rotarum," in *Commentationes Societatis Regiae Scientiarum Gottingensis, Classis Mathematicae,* 1781, t. IV, pp. 3-25, and *idem.*, "De Dentibus Rotarum," t. 5, 1782, p. 3.

24. Johannis Bernouilli, *Opera Omnia,* Lausanne and Geneva, 1742, t. III, "Lect. Hospitalii," XXII, p. 454.

Kaestner also studied the teeth of the rack and showed that its teeth must be epicycloidal to work properly with epicycloidal gears. He also began the study of the desirable length of teeth of the epicycloidal form. Finally, he considers the minimum pressure angle possible for certain given teeth and shows a value of approximately 15° to be about right.

In showing a convenient method of describing the involute and how to apply it to the teeth of gears Kaestner used a principle later adopted by Ferguson,[25] Airy, and Willis.

By the end of the 18th century there was, then, adequate mathematics for both the epicycloidal and the involute forms to be applied scientifically to gear teeth. But this information was familiar only to mathematicians, was written largely in Latin, and in any case was hardly in a form which engineers of that day could use.

An interesting example of the state of the theory among practical men is James White's *Mémoire* of 1812.[26] White had applied Robert Hooke's spiral gear of 1666[27] to the bevel gear to produce a helical gear,[28] "roues coniques, sur lesquelles (au moyen des machines nouvelles et appropriées)[29] je trace et creuse des dents hélico-spirales." White says that these gears engage perfectly whatever the relation between the diameter of the wheels or the angle which their axes form with each other. At any rate, he describes as proof an experiment in which he turned such gears for several weeks at considerable speed and load, continually oiling

25. *Ferguson's Lectures on Select Subjects,* new ed. with additions by David Brewster (ed.), 2nd ed., Edinburgh, 1806, pp. 210-226.

26. James White, *Mémoire,* Paris, 1812.

27. Robert Hooke, *Lectiones Cutlerianae,* London, 1679, No. 2, "Animadiversions on Helvius 'Machina Coelestis'" pp. 70-72 and Figs. 20 and 21 (the date of 1666 is Hooke's).

28. Leonardo da Vinci had conceived such gears. See Codex Atlanticus, fol. 396Re.

29. White does not describe these machines in his *Mémoire,* but they are described and illustrated in his *New Century of Inventions,* Manchester, (n.d., *ca.* 1824).

them with a mixture of oil and emery! He says that the wear on the teeth *à l'endroit des cercles primitifs* was imperceptible. This seems hardly consistent with his repetition of the old false notion that his gear teeth will "wear in" and that there is therefore no need to attempt the epicycloidal form—any of several forms will "wear into" a constant motion form.[30]

Clearly an engineer as clever as White was badly in need of sound mathematical analysis of the action of gear teeth. So were most engineers and mechanics of his day.[31]

The Translators

We come then to the three men who were to make the transition from the mathematicians to the engineers and thereby to make possible the gears and gear-cutting machines of the later half of the 19th century—Hawkins, Willis and Buchanan.

The first step in this direction was John Hawkins' publication of a translation into English of Books X and XI of Camus.[32] This was a step in the right direction, had not Hawkins, in his zeal to make the mathematician's work easily available to the mechanicians, made the mistake of adding to his translation parts of the new edition of Imison, which unfortunately contained the erroneous statement that the proper generating circle of the epicycloid should be one with its diameter equal to the *diameter* of the opposite wheel, instead of equal to the radius.[33] This started a thirty-year controversy not worth considering here in detail, but it did have

30. He repeats this notion in his *New Century of Inventions,* (*ca.* 1824), p. 263.

31. See Hawkins below p. 22. Even a professor of mathematics at Heidelberg was content with empirical rules given by Dutch millwrights. See C. C. von Langsdorf, *Ausführliches System der Maschinenkunde,* Heidelberg, 1826. Langsdorf's rule was not very different from that given by Leupold one hundred years earlier!

32. John Isaac Hawkins, *Teeth of Wheels,* London, 1806.

33. Thomas Gill, (ed.), *Imison's Elements of Science and Art,* London, 1803, Vol. 1, p. 97; first edition was titled *The School of Arts,* London, n.d. (*ante* 1787).

two important results. First, many manufacturers took up the Imison method, and it was two generations before this error was corrected in practice. Second, the controversy whipped up in England a lively interest in the whole question of the form of gear teeth.[34]

Finally in his second edition of 1837 Hawkins rectified the error by citing numerous authorities (including Camus!) to show that Imison was wrong, and then in rather unmanly fashion put the blame for his error on a "friend of more than 30 years"—none other than Gill, the editor of Imison!

However, Imison should not be lightly dismissed, for it was he who first suggested the tooth form which has radii out to the pitch circle and from there has epicycloids to the ends of the teeth.[35] This was a form widely adopted in practice—perhaps because Imison gave a very convenient means for forming a brass templet for cutting teeth of this type for gears and racks.

But to return to Hawkins and his own original contributions. In his "additions" to the second edition of Camus of 1837 Hawkins points out the many advantages of shorter teeth. First, they reduce the amount of sliding friction. Longer teeth had been used to give the strength resulting from having more than one tooth engaged at a time. He suggests that the strength can be more easily increased by giving greater breadth to the teeth faces. Hawkins showed that sliding is only eliminated between identical gears and always exists otherwise; yet it can be reduced by the use of shorter teeth. He shows just how the amount of sliding can be determined geometrically in each case. Hawkins also notes that short teeth actually add strength since it is not necessary to cut back the base of the teeth to give the clearance required for longer teeth. Therefore the teeth can be made thinner for equal strength, which permits the use of a greater number of teeth on a given wheel, and thus the strain is more equally divided.

34. See David Brewster's comments in his second edition of Ferguson's *Lectures on Select Subjects,* Edinburgh, 1806, 1st ed., London, 1803; Thomas Young, *Lectures on Natural Philosophy,* London, 1807; also Buchanan below.

35. John Imison, *The School of Arts,* p. 33.

Hawkins showed that use of the diameter instead of the radius, as Imison advocated, leads to weakening the tooth by requiring cutback for clearance at the radial base of each tooth. Since these clearance indentations were not in practice made in the teeth by the millwright, the teeth were worn to provide them. Practical men had assumed that therefore the gear has been "worn in" to the proper form, so they simply copied the worn form when laying out a new gear.[36] But Hawkins demonstrated that, if the epicycloidal teeth are generated by the radius rather than the diameter, no such wear can occur.

Hawkins notes that the use of Hooke's principle of 1666 for spiral gears can eliminate the shocks arising from wear of bad figures of teeth. He then goes on to point out other errors in Imison, especially in regard to the teeth of racks.

At the very end he says: "Let him, however, who would go to work with an understanding of his subject, investigate for himself, and take nothing upon trust, but let him ascertain the truth of every proposition he admits and not blindly follow the practice, or submit to the judgment of others." Good advice to the scientist, the engineer and the historian from one who had learned it the hard way.

Having thus purged himself of his Imison sins, Hawkins then goes on to make a most significant contribution—to point out the value of the involute tooth as compared to the epicycloidal. This marks the real turning point in gear design, even though it was to be another generation before Brown and Sharpe began to make it widespread in practice.

Here[37] Hawkins admits that many others had thought of the possibility of the involute form,[38] but the epicycloidal form or its modifications had proved so generally satisfactory that until Hawkins no one had seriously considered its possible disadvantages as compared to the involute form, despite the fact that mathematical techniques, as well as basic principles of gear design, were already more than adequate for such an analysis.

36. See White, *New Century of Inventions*, p. 263.
37. Hawkins' original edition of Camus of 1806.
38. Euler was apparently the first in his "De Figura Dentium Rotarum," *Ac. Sci. Imp. Petro. Nov. Com.*, XI, 1765, p. 207.

Hawkins was led to the involute by considering the gear teeth required when a gear engages more than one other gear at a given time. The screw-cutting lathe and other machine tools raised this as a practical problem at just this time, especially for those machines which had "change gears," which had to be interchangeable. It was at once evident to him that the involute was far superior for this purpose because one involute gear of a given pitch can work with another of any size, but of the same pitch—except for pinions of few teeth. But Thomas Young had pointed out that "If the face of the teeth, where they are in contact, is too much inclined to the radius their mutual friction is not much affected, but a great pressure on their axes is produced and this occasions a strain on the machinery, as well as an increase of friction on the axes."[39] Young had deduced this result, not measured it.

Upon a suggestion from Joseph Clement,[40] Hawkins tried this theory out for various degrees of engagement of the teeth and resulting pressure angles up to 21°. He found no such force to exist in appreciable quantity. Hawkins correctly explained this as the result of the friction of the sliding teeth counteracting the force of separation, at least up to pressure angles of 20°. This meant, of course, that the distance between centers of involute gears need not be as accurately established as for epicycloidal—a great convenience for the millwright.

Hawkins sums up the other advantages of involute teeth: (a) In epicycloidal teeth the space must equal the tooth, but in involute teeth only a little more than one half the space is required for the involute tooth of proper length to enter; therefore a greater number of teeth of equal strength can be used. Because gear trains are usually designed to be reversible this principle has seldom been applied in practice, however. (b) With involute teeth of proper design there will be more than one tooth engaged at a

39 Thomas Young, *Lectures on Natural Philosophy and the Mechanical Arts*, London, 1807, Vol. I, p. 175, and plate 15.

40 Note Hawkins' close connection with the man who was at the very center of machine tool developments of the time.

given time. Therefore the strain can be easily divided. (c) Sliding of one tooth on the other is diminished and rolling of tooth on tooth increased for involute teeth. The sliding for the involute tooth will be about one half that for a similar epicycloidal tooth.

Therefore involute teeth not only give convenience in properly meshing several gears together, but give stronger gears and less friction and wear.

Hawkins goes on to sketch briefly how Camus' principles could be applied to the teeth of bevel gears—either epicycloidal or involute. But he seems to be completely unaware of the difficult problem of actually cutting such teeth, with "the sides of the teeth—accurately formed according to straight lines, all meeting together on the common point of intersection of the axes of the two shafts carrying the engaged wheels." This problem had to await the invention of the "octoid" tooth by Bilgram in 1885.

Hawkins finally returns to the problem for which he had originally included the "additions" from Imison—a simple device for drawing the proper figure of gear teeth, now worked out not for the epicycloidal form, but for the *involute*. By the aid of a bit of watch spring he gives detailed instructions on how to lay out quite simply any desired gear with involute teeth.

Before we leave Hawkins we must note some valuable information which he gives us on the actual practice in forming the teeth of wheels in his day. He questioned foremen, pattern makers, and workmen, and examined means, instruments, and tools used in a number of distinguished firms of engineers and millwrights. The results are astounding. Some had only "thumbed out the figure." Most had the crudest of empirical methods, some of which were actually incorrect in principle and practice. A few claimed to base their work on Camus and used the methods of Imison. Even the best mathematical instrument makers, chronometer, clock, and watch makers mostly used their eye in aiming at a modification of the Lancashire bay-leaf pattern! In Hawkins' day only Saxton of Philadelphia (and later of London) had made an instrument for producing truly epicycloidal gear teeth.

Clearly Hawkins marks the beginning of the transition from the mathematicians to the practical men, but he is more than that—he is one of the great names in the story of gears.

With Robert Willis (1800-1875) we have a man, himself a mathematician, of special interest to us not only because he extended the systematic analysis of gears, but also because he put the theory in a form in which the engineers could use it,[41] as Robertson Buchanan was to put it in a form suitable for the mechanic and the millwright. In fact, on his title page Willis says that his book is "Designed for the use of students in the universities and for engineering students generally."

Willis' primary aim was given: "My object has been to form a system that would embrace all the elementary combinations of mechanism, and at the same time admit of a mathematical investigation of the laws by which their modifications of motion are governed. I have confined myself to the Elements of Pure Mechanism."[42] The parts of Willis' system of greatest interest to us here are given on page xxi, entitled "Synoptic Table of the Elementary Mechanisms." Under this main heading he places:

Division A—Rolling Contact—Directional Relation Constant.
- Class A—Velocity-ratio constant.
  - Rolling cylinders, cones, and hyperboloids.
  - General arrangements and forms of toothed wheels.
  - Pitch.

Division B—Sliding Contact—Directional Relation Constant.
- Class A—Velocity-ratio constant.
  - Forms of the individual teeth of wheels.
  - Endless screws or worms and their wheels.

41. Robert Willis, *Principles of Mechanism,* London, 1841. This included his earlier work "On the Teeth of Wheels," in *Trans. Inst. Civil Engineers,* Vol. II, 1838, pp. 89-112. Matschoss, p. 70, says that Willis influenced even the German textbooks of Salzenburg, Weisbach and Reuleaux, because Willis presented gear theory as a part of a systematic treatment of mechanism.

42. Willis, *Mech.,* pp. xii-xiii.

By this system Willis was able to include the mathematical study of gears in the more general science of mechanism and thus provide for a complete analysis of the gear. However, as Willis specifically states, he has excluded from his book all questions of dynamics and therefore he does not treat of the strength of gears.[43] It is significant that at about this time Saxon brought out the first gear-cutting machine based on a *generating* principle.[44] This machine required a science of mechanism to make it possible; previous methods of using only formed-tooth cutters could be empirical or based only upon knowledge of the required curve without any understanding of how it could be generated.

Imison had shown the way to the analysis of the bevel gear by the use of cones of intersection.[45] Willis elaborates this method and uses the hyperboloid of revolution for the analysis of the spiral gear and its special case of the worm and pinion. In fact, Willis was able to show that the bevel gear is a special case of the spiral gear with the distance between the axes equal to zero. He went further to prove that as this axial distance becomes greater the rolling action is less and less perfect. In the case of axes neither parallel or intersecting Willis uses in effect two pair of cones.

In his study of these typical gears Willis does not have to resolve the question of epicycloidal versus involute teeth. However, Willis gives a thorough analysis of the problem of tooth form, thereby including all that had been done before, now presented in enlightening and systematic form. He considers all cases of the epicycloid and reduces them to the general case. This had been known to de La Hire as a possibility, but his method was imperfect. Thomas Young[46] had the proper method, but did not work it out fully. The most

43. For the beginning of this question, in Willis' day, see below p. 32 for Tredgold and Fairbairn's work.

44. See p. 75 below.

45. John Imison, *Mechanical Power*, London, 1787, p. 36 and Fig. 11. It was Imison who first introduced the term "bevel gear" and speaks of it as a type already well known.

46. Young, Vol. I, p. 175.

general solution was that of Airy.[47] The problem: "Given the form of the teeth of one wheel, to find the form of another that they may work together correctly." Airy stated the solution and gave a mathematical proof that it applied for *any* gear tooth:

"That the mechanical effect which one wheel will produce upon another, may in all positions be the same, it is necessary that the line perpendicular to the surfaces of the teeth at the point of contact, intersect the line joining the centers at a fixed point, which divides that line into two parts, the ratio of which is the mechanical power. When this holds, the proportion of the angular velocities will be constant."

Willis comes to advocate the involute form from a study of the path of the point of contact and of the smallest number of teeth possible for spur gears, both external and internal, and for racks. This led him to consider the ideal working depth and addendum,[48] as well as the thickness of the tooth and the breadth of space. He also introduced the constant 14½° pressure angle for involute teeth. Willis selected 14½° because it had a sine of very close to ¼. Later this value was retained because it also coincided closely with the pressure angle usual in epicycloidal teeth. It is also the angle used for worm threads, too, making the straight-sided rack of the involute system correspond in angle, as well as in other proportions, with the worm thread. All this work was based upon pure mechanism.

The result is a clear indication of the complexity resulting from epicycloidal teeth, especially for the cast teeth common at that time—separate molds would be required for each gear if they were to fit each other. Willis recognizes the limitations of the epicycloid for an interchangeable system of

47. George B. Airy, "On the Forms of the Teeth of Wheels," *Cambridge Phil. Trans.*, 1825, Vol. II, p. 277.

48. He used an addendum $= \dfrac{1}{\text{diametral pitch}}$ and showed that this gave a convenient height of tooth.

gearing. The advantages of the involute form stand out in the greater strength of this form, especially as compared to the epicycloidal with radial flanks.[49] (Fig. 3)

Willis showed that backlash could be easily minimized with involute teeth simply by adjusting the center distances. This was a great advantage for the millwright.[50] Willis, however, repeats Young's belief[51] that the pressure angle of the involute tends to force the center apart. We have seen Hawkins' answer to this.

A study of the engagement of gear teeth with a rack led Willis to note that the teeth of the involute rack have straight sides and that the rack will be forced down by the pressure angle, resulting in less vibration. He also noted that contact is not at a single point on the involute rack tooth as with the epicycloidal. Because the involute bears on most of the rack tooth face it will give less wear.

Contributions to the theory of the worm and pinion were also made by Willis. After describing the endless screw of Pappus[52] as a worm and pinion (worm wheel), he considers the form to be given to these teeth. The question then arises, how to cut them? Willis suggests "making the screw cut the teeth."[53] This had been done before by Ramsden,[54] who had first cut a gear by using a hob in 1768. Willis also made some contributions to the controversy over the Hindley worm.[55] In his study of the double-and-triple-threaded worm Willis showed the worm and pinion to be the special

49. This type of tooth had been introduced by Imison because the epicycloidal flanks required an undercut which could be relatively easily made by hand, but could not be done with a milling type cutter. Teeth using hypocycloidal forms out to the pitch circle and epicycloidal forms from the pitch circle to the end of the tooth were proposed by Willis and used. The involute tooth had no such problem, but the epicycloidal form died hard.

50. See Hawkins, *op. cit.* (1837), pp. 99 and 100.

51. Young, Vol. I, p. 177.

52. *Pappi, Math. Col. Commandini,* Bononiae, 1660, lib. VIII, p. 461 and prop. 24, p. 480.

53. Willis, p. 163.

54. Jesse Ramsden, *Description of an Engine for Dividing Mathematical Instruments*, London, 1777.

55. Willis, *Mech.*, p. 164.

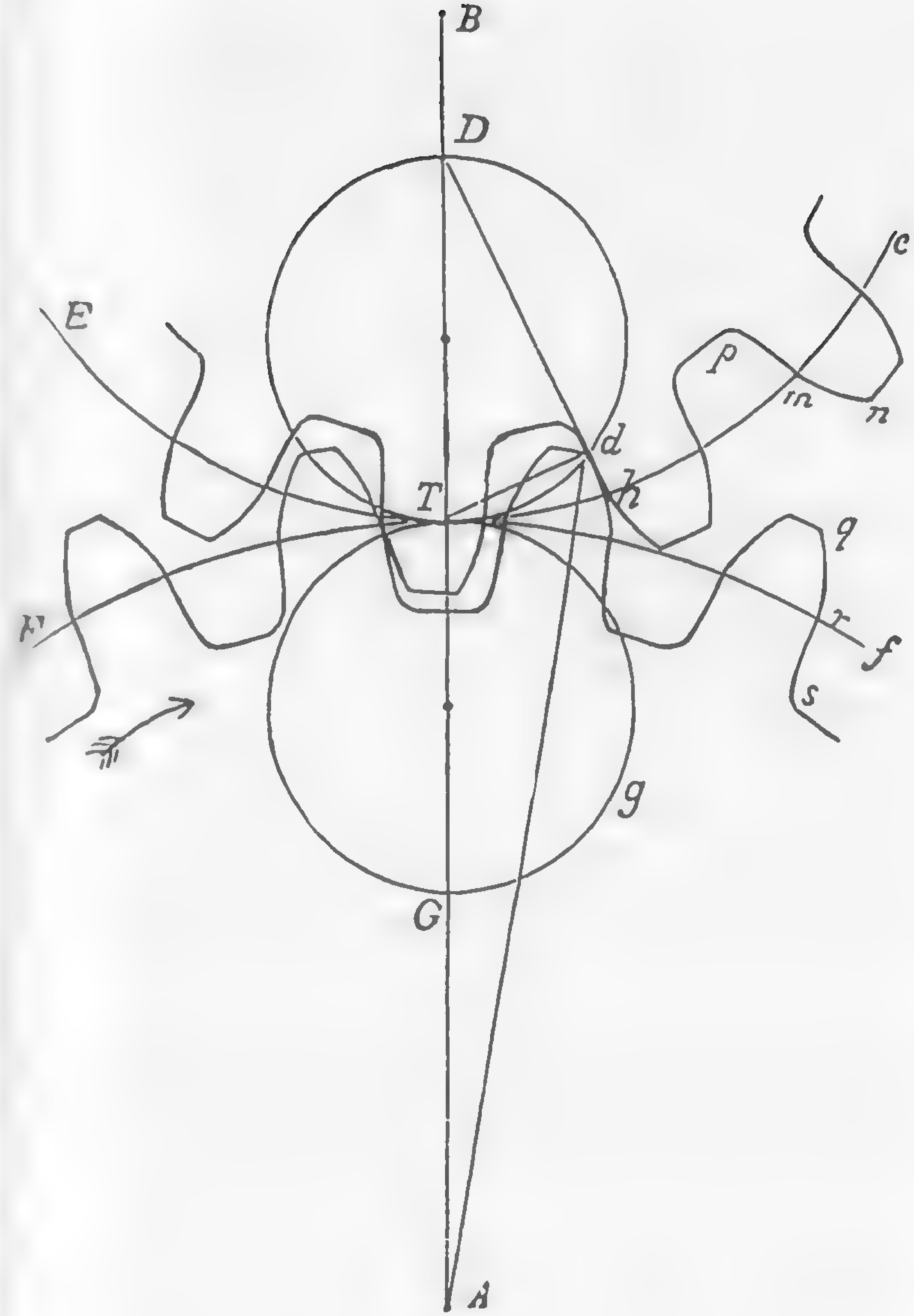


FIG. 3. WILLIS' INTERCHANGEABLE EPICYCLOIDAL GEARS, 1838 (*Willis*)

case of the spiral gear where the number of threads is one, two, or three. In this way he was able to provide a theoretical basis for the spiral gears of the Piedmont silk mill of 1724.[56] His was the first published account of the circular versus diametral pitch. The advantages of diametral pitch had been recognized by Bodmer.[57] This was called Manchester Pitch. Willis gave it its present name and listed values in common use in both systems.

It should be evident that Willis was far more than a mere systematizer; he made very substantial contributions of his own to gear theory.

But we must also examine his work in putting all this in a form in which the engineers would adopt it. In his earlier paper Willis had given a practical solution to the problem of laying out gear teeth. He took up the question of approximation to the involute in laying out teeth. Camus' rule-of-thumb method was the only one in use for laying out epicycloidal teeth by use of two circular arcs.[58] The theory of this had been worked out by Euler, but it had no practical effect in his day. Now, in 1838 Willis invented and named the first Odontograph (Fig. 4a, 4b), showed how to make one, and gave the necessary tables for the layout of involute teeth.[59] This device could also be applied to gear cutters; Willis shows how. He also indicates that a limited number of these cutters

56. See J. A. Borgnis, *Traité complet de Mécanique appliquée aux arts, Machines pour confectionnes des étoffes*, Paris, 1820, p. 160; also see Borgnis, *Dictionnaire Mécanique*, (Paris?, *ca.* 1820), Plate 31, Fig. 2; and J. M. L. de La Platière, *Encyclopédie Méthodique, Manufactures et Arts*, Paris, 1784, Vol. II, Article "Soie," p. 31 and plates 8 and 9.

57. J. G. Bodmer, "On the Pitch of Spur and Bevel Wheels and the Shape of the Teeth of Worm Wheels and Worms Working into Each Other," listed in *Min. and Proc. Inst. Civil Engineers*, 1843, Vol. II, p. 32. Bodmer based his upon a metrical system.

58. See Imison, *School of Arts*, and Andrew Gray, *Experienced Millwright*, Edinburgh, 1804, p. 17 and Plate II. Gray still uses the "rule of seven" for tooth proportions.

59. They were widely adopted in factories. These were sold by Brown and Sharpe for many years from 1852. See Robertson Buchanan, *Treatise on Mill Work*, London, 1841 (3rd ed.), p. 150, and Brown and Sharpe catalogue for Jan. 1, 1875.

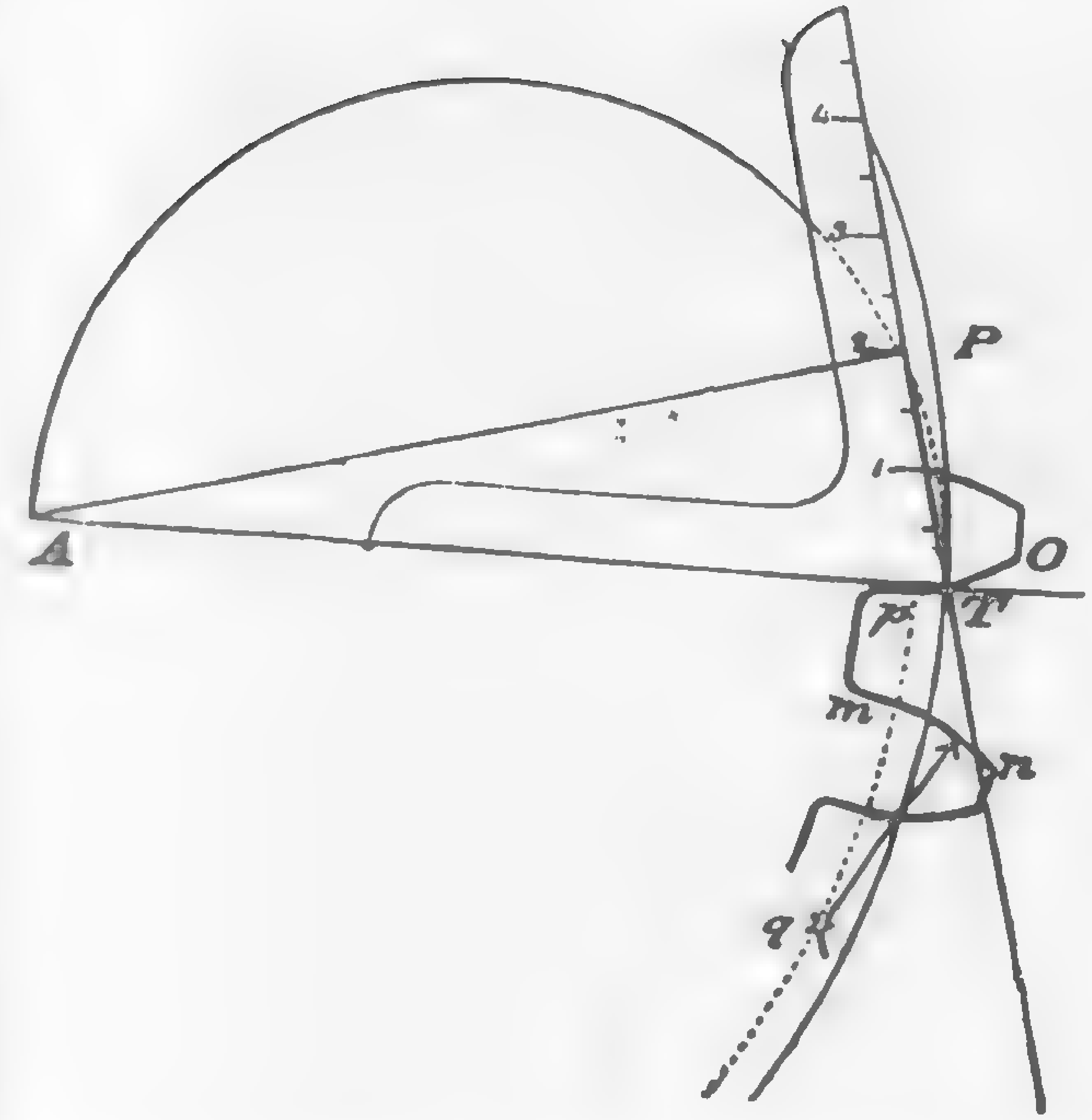


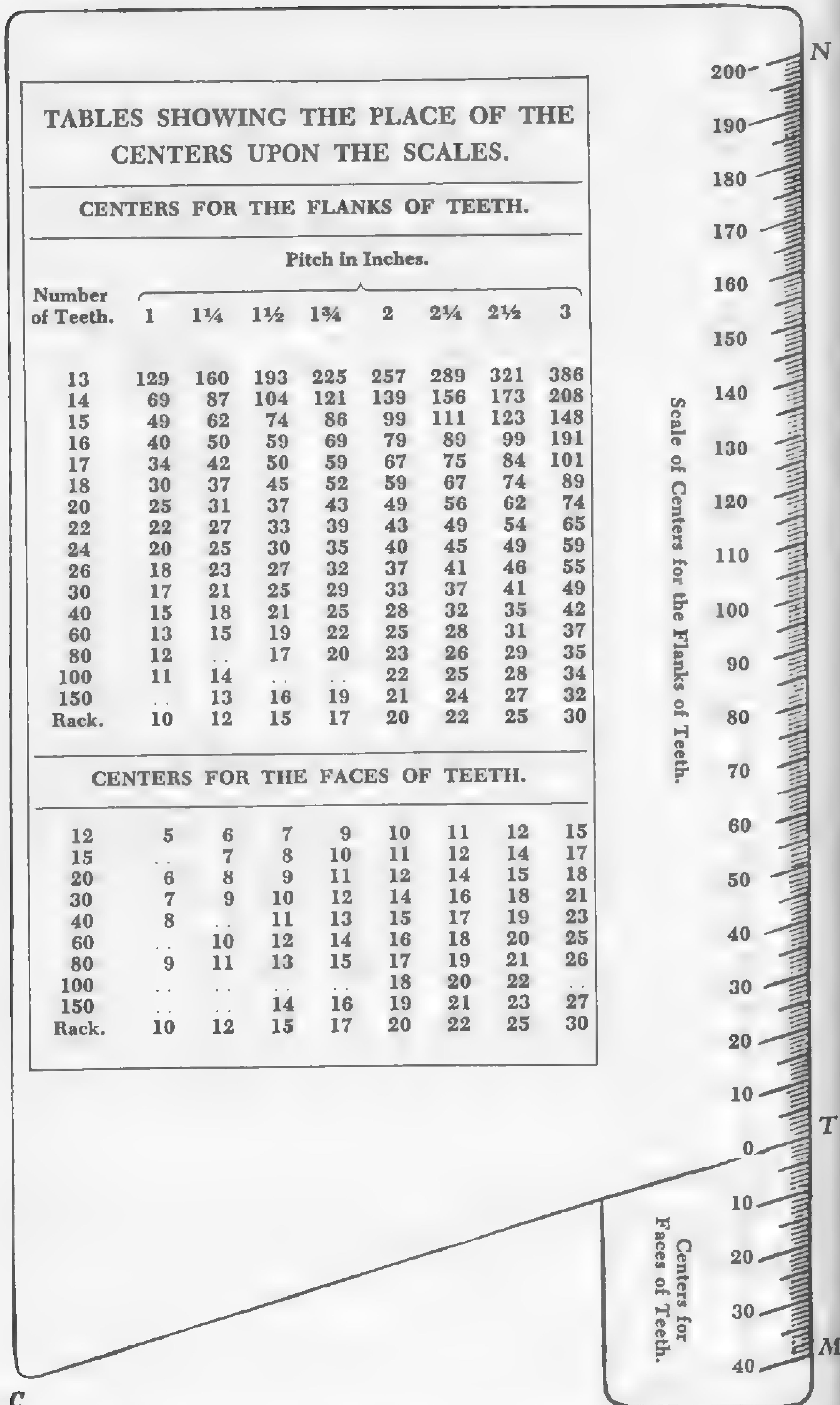
FIG. 4A. PRINCIPLE OF WILLIS' ODONTOGRAPH (*Willis*)

are required to produce involute teeth as compared with epicycloidal. In fact, he gives the first list of sizes which will make all common gear teeth within tolerances.

Willis even gives an approximate design for Hooke's helical gears of 1666.[60] But more important was his analysis, for the teeth of bevel gears, of an idea first suggested by Tredgold[61] in which the conical tangent surfaces are developed into planes. This was done for epicycloidal teeth, al-

60. Willis, *Mechanism*, pp. 126-135. These had been reinvented several times. See James White, *Century of Inventions*. Also see Timothy Sheldrake, *Theory of Inclined Plane Wheels*, London, 1811.

61. In Buchanan's *Essays on Millwork*, 1823, p. 103.



TABLES SHOWING THE PLACE OF THE CENTERS UPON THE SCALES.

CENTERS FOR THE FLANKS OF TEETH.

Number of Teeth.	Pitch in Inches. 1	1¼	1½	1¾	2	2¼	2½	3
13	129	160	193	225	257	289	321	386
14	69	87	104	121	139	156	173	208
15	49	62	74	86	99	111	123	148
16	40	50	59	69	79	89	99	191
17	34	42	50	59	67	75	84	101
18	30	37	45	52	59	67	74	89
20	25	31	37	43	49	56	62	74
22	22	27	33	39	43	49	54	65
24	20	25	30	35	40	45	49	59
26	18	23	27	32	37	41	46	55
30	17	21	25	29	33	37	41	49
40	15	18	21	25	28	32	35	42
60	13	15	19	22	25	28	31	37
80	12	..	17	20	23	26	29	35
100	11	14	..	..	22	25	28	34
150	..	13	16	19	21	24	27	32
Rack.	10	12	15	17	20	22	25	30

CENTERS FOR THE FACES OF TEETH.

12	5	6	7	9	10	11	12	15
15	..	7	8	10	11	12	14	17
20	6	8	9	11	12	14	15	18
30	7	9	10	12	14	16	18	21
40	8	..	11	13	15	17	19	23
60	..	10	12	14	16	18	20	25
80	9	11	13	15	17	19	21	26
100	..	..	..	..	18	20	22	..
150	..	..	14	16	19	21	23	27
Rack.	10	12	15	17	20	22	25	30

FIG. 4B. WILLIS ODONTOGRAPH AND TABLES, 1838 (*Willis*)

though he considers, not very fully, the bevel gear with involute teeth. Perhaps he recognized that these would have to be very thin wheels and so not too useful.

With Willis, then, the *geometry* of the common gears had been well worked out into a system. Gear design had been put into a form which engineers could understand and use. Much remained to be done in detail, but the only great question of theory remaining was that of tooth form. Practice was of course much slower.

In his "Essay on the Teeth of Wheels"[62] of 1808 Robertson Buchanan (1770-1816) claims only to have put the work of Camus and de La Hire in a form that "those who do not possess the advantage of a mechanical education" can utilize them. Taken with his tables for design, good gears would be produced in practice. In the third edition of 1841[63] the editor George Rennie has added a set of shop rules, based upon Willis' paper of 1833, for laying out epicycloidal teeth by using arcs of circles to give a sufficient approximation. Willis' odontograph and tables would do for the engineer, but the shop hand needed some rules and tables he could follow using only his familiar compass and scale as tools. Buchanan provided them in simple form.

With the work of Willis and Buchanan the engineers and the shop men had the means to produce mathematically designed teeth and by the 1840s were beginning to be convinced of the need for them. Two practical questions remained to be settled: "epicycloid or involute?" and "what about strength?"

62. Robertson Buchanan, *Essay on the Teeth of Wheels*, Edinburgh, 1808.
63. Often appended to Buchanan's *Treatise on Mills and Millwork*.

The epicycloid-involute controversy had been settled in theory by Hawkins and Willis about 1840, but was not to be decided in practice for another generation and a half. The strength of teeth and of gears generally first became of significance in the 1820s, required eventually the more refined tools of analytical mechanics for a solution, and finally a retreat from geometric perfection in order to achieve mechanical perfection. Toward the end of the century a new method of making gears, hobbing, introduced new questions of theory. By 1910 there was a strong movement to standardize gears. And the automobile and the steam turbine brought new types of gears whose theoretical problems had to be solved. Perhaps most important of all, the existing theory had to be put in terms of the basic mechanical elements, the straight edge and the circle, in order to make possible gear-tooth *generating* machines. In all this work three great names stand out, Edward Sang, George Grant, and Oscar Beale.

In the days of wooden gearing the speeds and the loads were so low that the strength of the gears was only an empirical problem hardly otherwise solvable in terms of the various kinds and conditions of wood used.[65] With the appearance of cast-iron gears used for higher speeds and higher loads and having somewhat more uniform material characteristics, there was a need to consider the strength of the gear, and a more systematic approach was possible. A beginning was made in 1822 by Tredgold.[66] The 1841 edition of Buchanan[67] contained a very elaborate account of the strength of teeth, both wood and cast iron, of the principal types, with tables and even a graph for proper design of teeth of adequate strength. But the practice remained largely empirical. In 1864 Fairbairn[68] was content to compare the practice of his day with Tredgold's theory and to discover with pleasure that both gave about the same results. Even with Reuleaux,[69], although much more mechanical analysis had been done, the science of testing materials was so little developed that he had to depend on largely empirical data for his constants. The beginning of a genuine analytical mechanics of gears is to be found in the work of Stribeck[70] in 1894. It culminates in the brilliant work of Earle Buckingham of M.I.T.[71]

64. The phrase was first applied to Oscar Beale of Brown & Sharpe.
65. See Leupold, p. 40.
66. Thomas Tredgold, *Practical Essay on the Strength of Cast Iron*, London, 1822, pp. 85-86.
67. Robertson Buchanan, *Treatise on Millwork*, 3rd ed. London, 1841, pp. 83-172.

After 1841 the epicycloid-involute controversy continued. Willis had provided a means by which epicycloidal teeth could be used for gear trains, and various others were suggested. Even down until the 1880s the epicycloidal form was dominant. Professor C. W. MacCord's epicycloidal engine was described in the *American Machinist* for August of 1880. And even Oscar J. Beale's odontograph of 1876 and his odonton engine produced Willis' double epicycloidal teeth.

The two opening salvos in the last battle of the War of the Gear Teeth were fired by George B. Grant—his *Handbook of the Teeth of Gears,* Boston, 1885, and his *Odontics, or the Theory and Practice of the Teeth of Gears,* Lexington, Massachusetts, 1891. Grant notes[72] that even in 1885 epicycloidal gears were the most common in use, especially for heavy gearing and for clock and watch gears, but some firms still used empirical approximations. Some used the interchangeable epicycloids with radial flanks. Grant's mathematical analysis sets up an expression for the relative efficiency of the epicycloidal versus the involute, and he is able to show that:

68. William Fairbairn, *Treatise of Mills and Millwork,* London, 1864.
69. Franz Reuleaux, *Lehrbuch der Kinematik,* Braunschweig, 1875.
70. R. Stribeck, "Berechnung der Zahnräder," in *Z.V.D.I.*, 1894, p. 1182.
71. Earle Buckingham, *Analytical Mechanics of Gears,* N. Y., 1949. His methods replaced the Lewis design equations, and he was awarded the Gold Medal of the American Society of Tool Engineers in 1957 for this work.
72. See also his articles in the *American Machinist,* September 1883, p. 5, and December 1885, pp. 2-3.

1. The epicycloidal is *always* less efficient than the involute.

2. The gain in efficiency in using the involute increases as the number of teeth in the base gear of the interchangeable epicycloidal system decreases.

3. For the stepped gear the involute is always more efficient, and for the spiral gear there is no difference.

4. For internal gearing the involute is always more efficient, and the gain in efficiency increases as the two gears approach the same size.

5. The friction varies as the square of the circular pitch, and therefore strength is best gained by increasing the size of the tooth.

Grant concludes[73] that the involute tooth is superior in adjustability, uniformity of pressure, friction, thrust on bearings, strength, and even in appearance! The only possible exception is in pinions of very few teeth. And that "the common opinion among millwrights and the mechanical public in general, in favor of the epicycloid, is a prejudice that is founded on long-continued custom, and not on an intimate knowledge of the properties of the curve."[74]

Grant's theoretical artillery required, however, the support of Brown and Sharpe infantry actually to take the ground of practice. This attack goes back to Joseph R. Brown's invention of his formed gear cutter in 1864.[75] Brown and Sharpe brought these out in epicycloidal sets of 24, sufficient to cut all gears of a given pitch from a 12-tooth pinion to a rack.[76] It was noted[77] that the involute system required only 8 to do equally satisfactory work, and that by bringing out sets of 15 for involute teeth a very high degree of accuracy in gear cutting could be obtained. It was also

73. Grant, *Handbook,* p. 20.

74. Grant, *American Machinist,* Dec. 1885, p. 3.

75. Brown's patent No. 45,294 of November 29, 1864.

76. They are listed in the earliest Brown and Sharpe catalogue, May 15, 1867.

77. Tables computing such sets for diametral pitch from 4 to 25 are in Brown and Sharpe files dating from July and August 1864.

noted that the involute cutter had less tendency to "drag" than the epicycloidal. Brown and Sharpe brought out in 1867 such sets, including diametral pitch. Since at this time Brown and Sharpe was the only firm making gear cutters for the market, the already great prestige of this establishment weighed heavily in the adoption of involute teeth in practice. By 1898 a survey by the *American Machinist* indicated very wide acceptance of the involute tooth.[78]

The more extensive use of helical and herringbone gears in the automobile and as reduction gears for the steam turbine raised some special theoretical problems. The principal problem that was new was the end thrust of the helical gear with these higher loads. Charles H. Logue showed[79] how to reduce this to a minimum by the choice of the angle of the helix to provide for continuous engagement of the teeth. Of course the herringbone gear had no end thrust as a whole, but the thrust was still there on the engaged teeth. This type of gear led to the development of a special type of machine to cut them—the Wüst.[80]

After 1900 the use of various types of helical gears in automobiles led to doubts of the desirability of geometrically correct gears. Gears had been brought to a technical perfection greater than the bearings which supported them, so that when their axes were thrown out of line their exact teeth no longer engaged properly. It was therefore necessary to design teeth to give smooth running under the conditions of slight misalignment. In his patent of 1904 Hugo Bilgram showed[81] that the noise of gears at high speeds was a question of transfer of the load—an alternation of one tooth taking suddenly just one-half the load, then suddenly the whole load. Bilgram designed teeth to produce a gradual taking and release of the load. In 1902 Eberhardt used a gear-generating hob to

78. *American Machinist,* March 1898, p. 193.

79. In *American Machinist,* Oct. 1907, pp. 573-575.

80. See Percy C. Day, "Herringbone Gears," *Journal A.S.M.E.,* 1912, pp. 75-104; Kammerer, *Entwicklung,* pp. 260-266. Also Charles Augustus, in *Machinery,* Dec. 1910, p. 270; and Percy C. Day, *Machinery,* Jan. 1911, p. 384, and N. Leerberg, *Am. Mach.,* 1922, p. 597.

81. Patent No. 749,683 of Jan. 12, 1904.

get this same effect. By 1921 the demands of automobile gears had produced a spiral bevel gear having its tooth-bearing surfaces relieved at the large and at the small ends of the teeth to give smooth operation even under slight shaft and bearing deflections. Straight-tooth bevel gears were being used in which the tooth length was one-quarter the cone distance, rather than one-third to one-half. Many other variations and special forms were being advocated.[82]

Back in the 1880s Grant had put forward suggestions of the value of more standardization of gears[83] and a few people had agreed from time to time. The first organized effort, however, in the direction of standardization grew out of Ralph Flanders' paper of December 1908, before the American Society of Mechanical Engineers.[84] This gave rise to discussion by practically every gear authority in the country.[85] Throughout 1910 the journals are full of articles on this question of standardization of gears. The ASME had appointed, in January 1909, a committee, with Wilfred Lewis as chairman, to prepare recommendations for consideration at joint meetings with the British Institute of Mechanical Engineers.[86] This committee had in October 1909 sent out a questionnaire to manufacturers of gears and gear cutters asking for their opinions. About one hundred replies were received, many conflicting. The results were presented to the joint meeting in Birmingham, England, in July 1910.[87] J. D. Stevens presented another proposed standard to this same joint meeting. There was no general agreement, especially on the stub-tooth gear, but the general consensus was to use the "Brown and

82. Henry J. Eberhardt, *Influence of the Automobile on Gear Cutting and Gear Cutting Machinery*, meeting Am. Soc. Mech. Eng., May 26-27, 1921, partially reprinted in *Mechanical Engineering*, Aug. 1921.

83. Grant, *Am. Mach.*, Sept. 15, 1883, p. 5.

84. Ralph Flanders, "Interchangeable Involute Gear Tooth Systems," *Journal A.S.M.E.*, Dec. 1908, pp. 1501-1520.

85. Wilfred Lewis, "Interchangeable Involute Gearing," *Journal A.S.M.E.*, Oct. 1910, p. 1631, and *Am. Mach.*, 1909, pp. 307-314.

86. *Am. Mach.*, Aug. 1910, p. 305.

87. *Journal A.S.M.E.*, Oct. 1910, p. 1631, with detailed criticism by both American and British authorities.

Sharpe standard."[88] The Committee submitted a majority report at the Spring Meeting in Baltimore 1913 of the ASME.[89] These results are significant primarily because they are the first standards proposed on the basis of experimental research on gears. The research was done at M.I.T. by H. S. Waite and by Everett St. John at the Wilfred Lewis plant in Philadelphia—an interesting combination of university and industrial research.

Because of lack of general agreement, there the matter rested, without official sanction, but with Brown and Sharpe practice becoming more widespread. The unmodified 14½ degree involute gave too much undercutting in pinions of few teeth. Both Ralph Flanders and Oscar Beale suggested making the tooth shape radial below the involute base circle, thus giving epicycloidal tips to the interfering portions of the mating gears. By increasing the pressure angle to 20 degrees and giving the teeth a shorter addendum, Fellows' later stub-tooth system eliminated all these problems. Finally the question was taken up by a committee of the American Gear Manufacturers' Association,[90] which under the guidance of Earle Buckingham arrived finally at our present standards.

The year 1910 was an exciting one for gear men. Not only was standardization the subject of lively discussion, but this was the year of the great "hobbing controversy." Hobbing methods of gear cutting had become common in the 1900s. Points of theory arose to determine the exact shape of the resulting teeth. The question had been raised by Grant in his article "Why Gear Hobbing Machines Cut Flats."[91] The controversy spread to England[92] and to Germany;[93] with Grant stoutly upholding his views[94] and various Americans

88. With the dissenting voice of C. H. Logue in *Am. Mach.*, Dec. 1910, p. 1171, and reply by L. D. Burlingame in *Am. Mach.*, Feb. 1911, p. 267.

89. *Journal A.S.M.E.*, 1913, pp. 1405-1420.

90. *Machinery*, 1923-1924, p. 374. Letter from Ralph Flanders Dec. 30, 1957.

91. *Am. Mach.*, 1910, p. 1148.

92. W. G. Groocock, in *Am. Mach.*, Sept. 1910, p. 411; P. A. Thompson, in *Am. Mach.*, Dec. 1910, p. 1171.

93. C. Barth, in *Am. Mach.*, Nov. 1910, p. 978.

94. *Am. Mach.*, Sept. 1910, p. 421, and Dec. 1910, p. 1119 and p. 1219.

entering in.[95] The matter was finally cleared up by a brilliant series of experiments by Flanders.[96]

A consideration of the very important studies of Edward Sang (1805-1891) takes us back in time, but since it was he who made the fundamental transition from the mathematicians to the *generating* type of gear-cutting machine, we can best describe his contributions at this point. Sang's new approach to the problem of gear teeth was first announced in 1837 in a paper before the Royal Scottish Society of Arts.[97] After several revisions and extensions Sang incorporated this method in his book of 1852.[98]

The book was, as Sang's title indicates, a *general* theory of gear teeth. Using the calculus and analytic geometry, Sang develops the theory in elegant form. He insists upon the desirability of sets of interchangeable gears. From a consideration of gears having more than one point of contact at a time he arrives at the principle of the "hour-glass curve"—the locus of the tracing point.[99] This very convenient and general method enabled Sang to consider in general terms the question of the minimum number of teeth on a pinion, as well as the relative claims of the involute and the epicycloid. Sang arrived at a tooth form of his own as optimum not only geometrically but from minimum effect of wear on the action of the teeth.[100] For the design of these teeth he provided the necessary tables, given to an accuracy of ten-thousandths of an inch. His system involved a varying pressure angle—16° 49′

95. J. E. Sweet in *Am. Mach.*, Oct. 1910, p. 741; F. O. Farwell, *Am. Mach.*, Nov. 1910, p. 867.

96. R. E. Flanders, *Am. Mach.*, Dec. 1910, p. 1064, and *Machinery*, Jan. 1911, p. 369, Mar. 1911, p. 569, June 1911, p. 798; H. H. Asbridge, *Am. Mach.*, Dec. 1910, p. 1211; E. J. Less, *Machinery*, Apr. 1911, p. 659; G. B. Grant, *Machinery*, June 1911, p. 813.

97. Edward Sang, in *Trans. Roy. Scot. Soc. Arts*, 1837.

98. Edward Sang, *New General Theory of the Teeth of Wheels*, Edinburgh, 1852.

99. Later used by Grant in analyzing, in a much simpler way, the teeth of bevel gears to give his "octoid" teeth.

100. Seldom adopted in practice and later criticized in theory by Grant in *Am. Mach.*, 1898, pp. 408 and 486.

to 24° 09′.[101] Sang also anticipated Grant in using the term "odontoid" for the optimum curve for the teeth of a given gear.

Sang points out that gear teeth may be designed not only for interchangeability and proper action of the gears with each other, but also for minimizing friction, for ease of manufacture, and for other considerations such as strength or inaccuracy of center distances. He analyzes gears in simple yet broad mathematical terms for minimum friction and wear effects and for ease of manufacture and thus laid the foundation for general analytic treatment of gears from these other considerations.[102]

A detailed mathematical analysis of the design of gears for minimum friction is given and also for minimum effect of wear on their operation.

It is Sang's mathematical analysis of the problem of manufacture of gears that interests us most at this point, for the all-important relation between theory and practice had hitherto been ignored, and Sang was the first to make the *transition from geometry to metal* upon which all successful gear-generating machines since his time have been based.[103] Sang classifies the "entomy of wheels" under four heads:

1. The formed cutter—rotary, broach,[104] or single-point tool.
2. The rack cutter and the generating pinion.
3. The generating circular cutter following a calculated curve.
4. The generating cutter following the combination of the tracing point and the angular motion of the wheel.

101. Kammerer incorrectly credits Hoppe with being the first to use varying pressure angles, but Hoppe's work was not done until the 1870s.

102. This work was later extended by Stribeck, *op. cit.*

103. The problem was further studied by: Th. Olivier, *Théorie géométrique des engrenages*, Paris, 1842; Gustav Herrmann, "Die Zahnflächen und ihre automatische Erzeugung," in *Ver. der V. Beförderung des Gewerbfleisses im Preussen*, Berlin, 1877, p. 61; K. Kutzbach, "Gundlagen und neure Fortschritte des Zahnräderzengung," in *Z.V.D.I.*, 1924, pp. 913, 1076, 1105.

104. This method, Sang tells us, had been first introduced by Joseph Whitworth, but see below p. 49 for Polhem and p. 48 for Leupold.

Of these methods of gear cutting only the first two became of practical importance. The last two, as Sang describes them, were of significance only as they led him to a more general mathematical analysis of the whole problem.

For the rotary-formed cutter, for example, he indicates original expense, difficulties of making and sharpening the cutter, need of a set for each pitch and for each diameter—an enormous collection. He notes that all formed cutters—rotary, broach or single-point—require checking by a templet and so describes his "miglioscope," the first gear comparator. He shows how this device may also be used for proper alignment of the tool with the axis of the gear blank. Sang describes the limitations of the clock-makers' index wheel and advocates a "snail-index-wheel" which uses an accurately cut worm. Sang had clearly learned from the instrument makers.

The use of the rack cutter is also treated in general terms and as a practical problem. Sang points out that any rack tooth desired can be used and that all wheels of a given pitch are then cut by a single tool. He shows us how to compute curves for the ends of the teeth of the involute rack. The machine can be easily made self-feeding, but this process cannot of course be applied to internal gears. Sang thinks the practical problems of the generating pinion method outweigh its usefulness. Fellows was to show how it could be done in practice.

In all these methods Sang notes that the resulting teeth are dependent upon the accuracy of the forms of the cutter (though he failed to note the ease of getting this accuracy in the involute rack). In practice in his day it was possible to get a really true hardened cutting edge only on a straight edge (by a flat lap) and a circle (by grinding while turning in a lathe). Sang says, "But the straight edge will not answer to our purpose as it cannot be applied to the concave parts of the tooth. . . .[105]

Sang introduces, for his third method, a fixed circular cutter of radius less than any radius along the tooth, which he proposes, as one possibility, to use in a very tedious process of computing and setting up coordinates of the odontoid by using microscopes. The other possibility is to carry the cutter along the path of the tracing point and keep its radius always directed toward the pitch point. Sang still proposes to do this manually by the micrometer; later the same result was achieved mechanically.

For the fourth method, Sang considers the mathematics of the two possible rectilinear motions of the tracing point—perpendicular to the line of centers, or obliquely through the pitch point. He then analyzes the circular motions possible. These general results are applied to the involute and to the epicycloid. He concludes that for the epicycloid the inner part of the tooth form must be radial and the outer part a truncated epicycloid. After long analysis Sang comes out to his own special tooth—a combination of his "kemend" and the "hour-glass," which he recognizes as far too complex for practice, but valuable for the breadth of treatment required.

And it was just this combination of a practical sense of the mechanical possibilities, with the most general logical and mathematical treatment of the problem of gear teeth that makes Sang's work, as is that of so many great men, the climax of all that had gone before and a transition to what was to follow.

105. Sang, p. 38.

II The Clock and Instrument Makers

THE FIRST GEAR-CUTTING MACHINES

REID, HINDLEY, AND REHÉ

THE INSTRUMENT MAKERS

The First Gear-Cutting Machines

Although even today many clock makers do not bother with exact tooth forms for their gears, long before the mathematicians had solved this problem the clock makers had actually produced satisfactory gears on gear-cutting machines. In so doing they solved many of the mechanical problems, and in particular that of accurate indexing. Because of the special numbers of teeth involved in clock work, this was a serious problem.

The earliest reference we have to a gear-cutting machine is found in Spain. Morales[1] describes the work of Juanelo Torriano (1501-1575) in constructing the great planetary for Charles V about 1540. Torriano had been born at Cremona in Lombardy and came to Spain, where he gained a reputation as a builder of aqueducts and of a "relox." His work is of such importance that I translate the relevant passage in Morales:

"It took him as he told me, all of twenty years to conceive and work out the plan; and because of the great amount of energy and concentration in thinking about it, he got sick twice during that period and almost died. Then having spent so much time in conceiving it, it did not take him more than three and a half years to make it by hand. This was quite a feat because the whole clockwork [relox] had more than 1800 wheels not counting many other parts of iron and brass that are involved. So every day (not counting holidays), he had to make . . . more than three wheels that were different in size, number and shape of teeth, and in the way in which they are placed and engaged. But in spite of the fact that

1. Ambrosio de Morales, *Las antigüedades de las ciudades de Espana*, Alcalá, 1575, pp. 91-93. I am indebted to my former colleague at the Smithsonian Institution, Dr. Derek J. Price, for this reference.

this speed is miraculous, even more astounding is a most ingenious lathe [torno] that he invented (and we see them today) to carve out with a file iron wheels [labrar ruedas de hierro con la lima] to the required dimension and degree of uniformity of the teeth. And in spite of all this and knowing that he did it all by hand, it is not surprising that Janelo says, as he does, that no wheel was made twice because it always came out right the first time. And if all he said were not better in actual fact, it would be very surprising."

Morales' description is tantalizing. We would seem to be safe in concluding that Torriano had a rotary file cutter not unlike those of two hundred years later, but since it cut iron it must have been hardened. One would give much for one look at it. At any rate, Morales indicates that Torriano's device became fairly common in Spain within twenty-five years. We should also like to know if he was really the inventor or if he brought the basic idea with him from Italy.[2]

The next textual reference we have to a gear-cutting machine is in Robert Hooke's (1635-1703) "Diary."[3] But we have as evidence of what Hooke's machine actually looked like only the brief description in Le Roy.[4] The Science Museum, London, has a wheel-cutting machine in its collections[5] (Fig. 5) which it dates as of about 1672. It is perhaps Hooke's; certainly very similar to his, for it is not inconsistent with the description of Hooke's machine in Le Roy. This machine has a fair index plate, a screw for adjusting the depth of cut, and gears of its own driving a formed cutter from a hand crank. For wide gears the base of the spaces between the teeth would be substantial arcs, since the cutting head only swings on pivots. However, it is probably the oldest gear-cutting machine extant. Although refined in detail,

2. A search in Leonardo da Vinci and his successors has failed to reveal anything definite, although Codex Atlanticus 294Va could be a gear-cutting machine with an index plate, and Inst. Fr. da Vinci MS. B, folio 71 has a drawing of what could be a rotary gear cutter with coarse formed teeth.

3. See references in R. S. Woodbury, *History of the Milling Machine*, The Technology Press, Cambridge, Mass., 1960.

4. Pierre Le Roy, *Étrennes Chronométriques*, Paris, 1758, p. 29.

5. SM Photo no. 719/55.

FIG. 5. WHEEL-CUTTING ENGINE, CA. 1672
(*Crown Copyright, Science Museum, London*)

there is no substantial improvement of design in the Science Museum's machine for cutting clock wheels of 1789[6]—a hundred years later.

Some advance is shown in France, which was to be the leader in other horological machines in the 18th century. In Nicholas Bion (1652?-1733) we find gear-cutting engines of 1702 shown[7] with formed rotary cutters. These seem to be less highly developed than the Science Museum machine of

6. SM negative No. 21753. Thomas Reid in Rees *Cyclopedia*, Article "Cutting Engine," says "present day" (1805) engines are not very different from the 1672 engine.

7. Nicholas Bion, *Traité de la construction et des principaux usages des instruments de mathématique*, Paris, 1709, pp. 97-100, and Planche X, p. 118, Fig. 1-5. English trans. by Edward Stone, *Construction and Principal Uses of Mathematical Instruments*, London, 1723.

1672, except that the cutter has a concave curve rather than a straight vee. In Leupold[8] we find a very similar machine, with a cutter square in cross section. But Leupold shows in his Fig. III a machine cutting by means of a straight broach. This is the first evidence we have of broaching gear teeth and the first machine to give flat bases to the spaces between the teeth. Leupold's machine also shows a curious arrangement for indexing, using a kind of divider.

In Antoine Thiout (1692-1767)[9] we have the first evidence of an horologist acquainted with the mathematics of gear teeth, for in Vol. I he reprints Camus' paper of 1733. He describes and shows[10] a "machine ordinaire" for cutting gear teeth. This machine has a support for the outer end of the gear blank spindle, but also has what seems an unnecessarily complex double frame to support the cutter spindle. Thiout also shows the machine of Henry Sully[11]—a doubtful improvement brought to England in 1711.

We can also find here a description of Fardoil's machine[12] with a worm drive and a ratchet click on the index, to give very large numbers of teeth. Thiout gives us a table from 102 to 800. The machine was the work of the Hulots, father and son. Reid[13] has grave doubts of the wisdom of such fine graduating when the basic 420 notches on the circumference of the dividing plate are of very doubtful accuracy.

A generation later we find very little advance in these clock-wheel "cutting engines." Ferdinand Berthoud (1727-1807) describes and illustrates one in detail[14] not very different from those in Thiout. It was probably the work of Hulot's son. In his *Horloges Marines* Berthoud describes a device

8. Leupold, p. 53, Tab. XV.

9. Antoine Thiout, *Traité de l'horlogerie*, Paris, 1747.

10. *Ibid.*, Vol. I, pp. 43-46, Plates 16, 17, and 18.

11. *Ibid.*, Vol. I, pp. 46-52, Plates 19-22.

12. *Ibid.*, Vol. I, pp. 53-56, Plate 23.

13. Thomas Reid, article "Cutting Engine," in Rees, *Cyclopedia.*

14. Ferdinand Bethoud, *Essai sur l'horlogerie*, Paris, 1786, Vol. I, Chap. 25 and 29, Plate 16. And Vol. II, Chap. 4, Plates 21 and 36. Similar machines are shown in his *Traité des horloges marines*, Paris, 1773, Arts. 1117-1128, and Plates 22 and 23.

for forming the ends of teeth by means of a concave file confined in a frame; the "cutting engines" cut only the sides and the base of the tooth. Berthoud is also one of the clock makers who knew of epicycloidal teeth; in fact, he shows how to draw them. In Diderot's *Encyclopédie* we find descriptions of wheel-cutting engines.[15] He describes and illustrates Sully's machine in detail. Of greater interest, however, is the engine of Hulot (son?) which has the first vertical slide for the cutting-tool arbor, and would therefore give flat bases to the spaces between the teeth. Hulot also added a fine adjustment for vertical centering. These French clock-makers' instruments were, however, largely anticipated by the wheel-cutting machines of that most remarkable Swede—Christopher Polhem. By 1729 he had developed a hand-operated gear-cutting machine of a production type, using reciprocating broaches as shown in Fig. 6. Soon after he had a power-

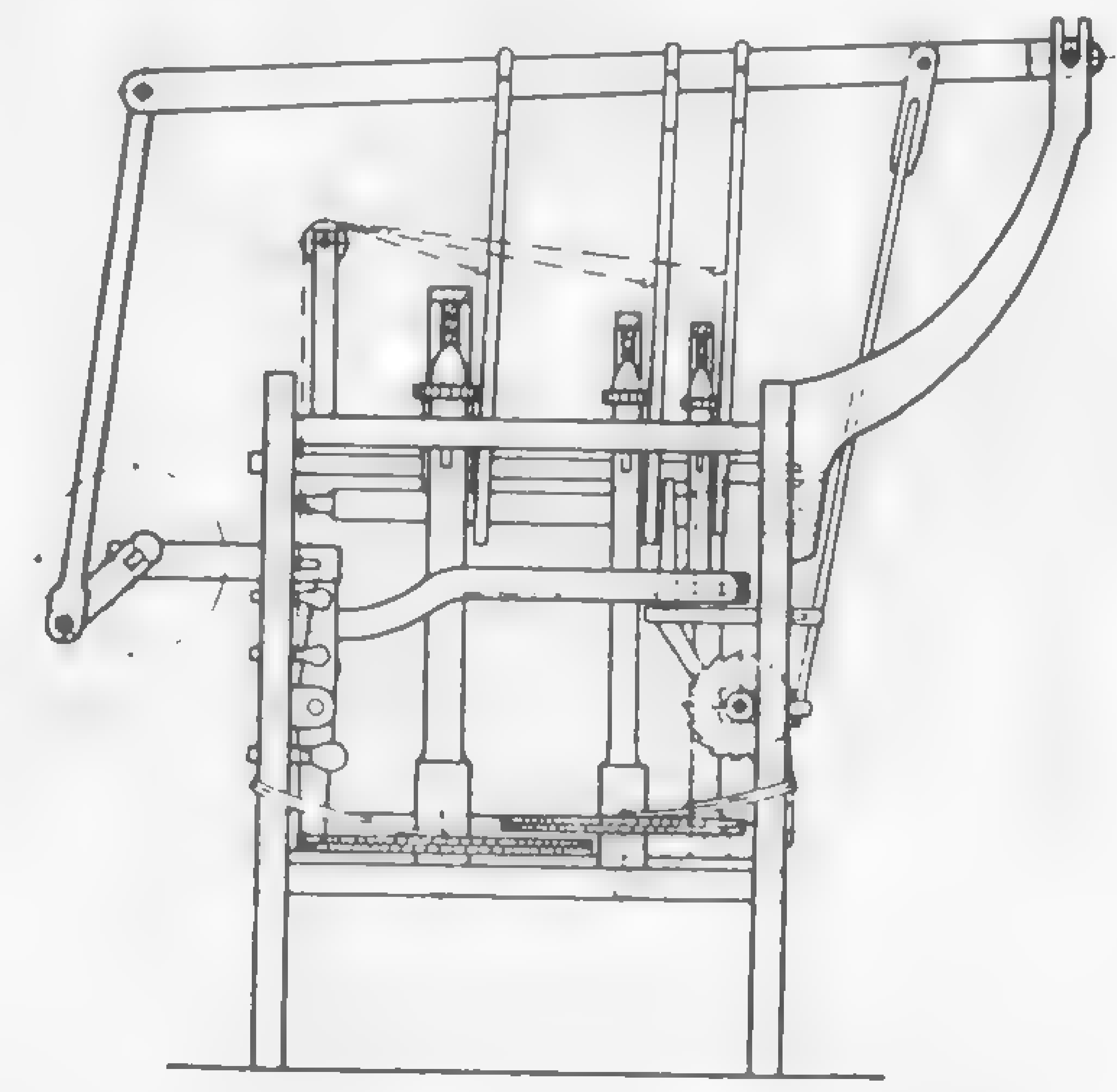
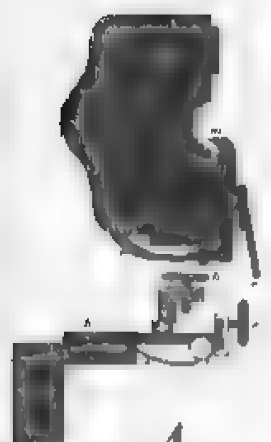


FIG. 6. POLHEM'S GEAR-CUTTING MACHINE, 1729 (*Matschoss*)

15. Diderot's *Encyclopédie*, Paris, 1777, Article "Horlogerie," t. VIII, and Article "Fendre (machine à)," t. VI. For Sully's machine see p. 486; for Hulot's, p. 483 and Plates 24-26.

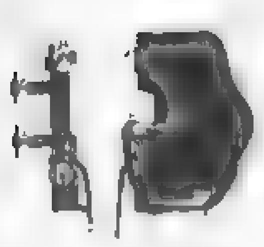
driven production machine using rotary cutters. These gear cutters formed part of a series of machines for large-scale production of clocks.[16]

Reid, Hindley, and Rehé



4.

The French clock makers are usually credited with great advances in clock-making tools in this period. But their cutting engines show little but refinement of the earlier English machines. For genuine technical advance we have to turn to Hindley and Rehé.[17] But we should first say a word about Thomas Reid (1710-1796) of Edinburgh, in whom we find combined the practical and skillful clock maker with the best knowledge of the theory of gears.[18] Reid had read Camus and so advocated epicycloidal teeth. He had calculated their values and compared his results to Berthoud's empirical values[19] with close agreement. Reid gives a table showing how to use his figures in determining the "geometrical diameter" and the size of the cutter. He also gives values for the addenda. This was the first attempt really to apply Camus in clock-wheel teeth, but even Reid still required some "fudging." He also corrected Imison in a very clear demonstration, more specific than that of Camus, to whom he gives full credit. Reid indicates that the "bay-leaf" tooth form (Fig. 2) was still common in his day, but he insists on the superiority of teeth cut by engines. He recognizes that the involute may be equally well used and refers to Brewster's edition of Ferguson, which points out that de La Hire had already noted



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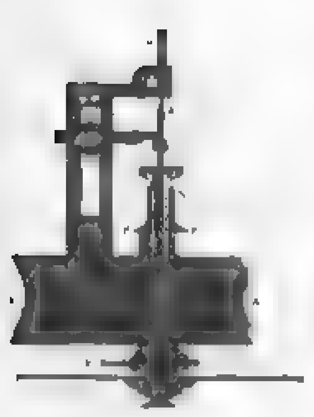


6.

16. Sten Lundwall, "Christopher Polhems Shärmaskiner för urhjul," in Särtryck ur Tekniske Museets Årsbok, *Daedalus,* 1949, pp. 52-62, with ten illustrations of contemporary drawings and machines in their collections. I am indebted to Dr. T. K. W. Althin, Director, Tekniske Museet, Stockholm, for a copy of this reference.
17. One exception is Taillemand's introduction in 1750 of a tubed arbor for the gear blank, instead of the arbor with a square hole. This produced much more accurate indexing.
18. Thomas Reid, *Treatise of Clock and Watch Making,* Edinburgh, 1826.
19. Berthoud, *Essai sur l'horlogerie,* t. I, p. 172.

the possibility of involute teeth. Reid sees, on the authority of "the late Professor Robinson," the advantages of involute teeth in permitting the use of many teeth acting at once, but he sees the calculation of these teeth as a matter of the calculus and therefore too complex! This is about as close as the clock makers ever came to the mathematicians in designing their gears.

The wheel-cutting engine of Henry Hindley (1710-1771) was first seen by Smeaton "in the autumn of 1741," but first described by him some forty years[20] later. As shown in Fig. 7 a number of valuable features will be noted at once.

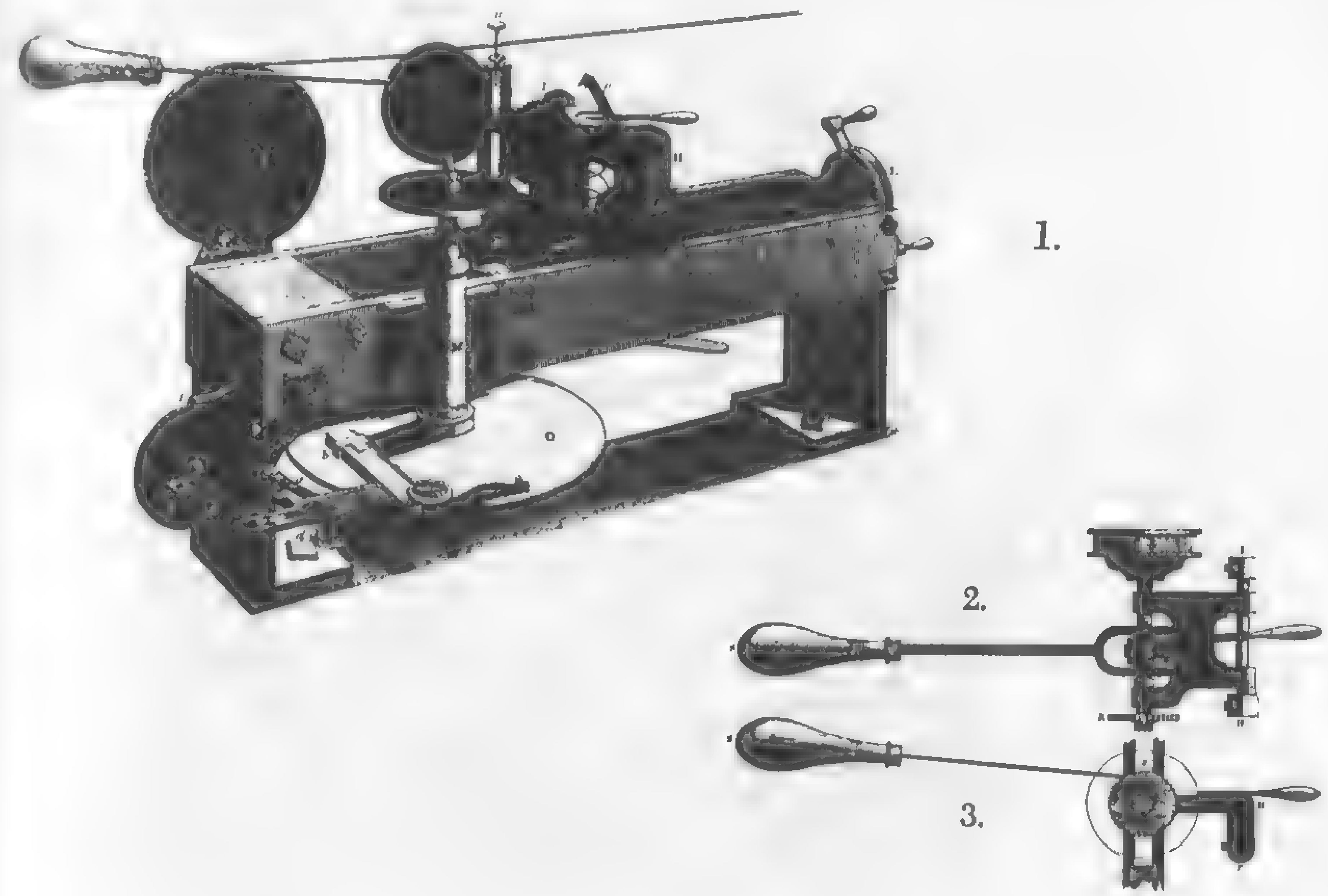


FIG. 7. HINDLEY'S ENGINE, WITH ATTACHMENTS, CA. 1741 (*Rees*)

20. John Smeaton, "Observations on the Graduation of Astronomical Instruments," in *Phil. Trans, Roy. Soc. Lond.,* Vol LXXVI, 1786, p. 1. The machine attributed to Hindley in the Castle Museum, York, is not even of his day. See *Horological Journal,* Nov. 1950, p. 725, and Dec. 1950, pp. 800, 803, 807.

Hindley uses a box-frame construction which would give far greater strength and rigidity than any of the French machines. The micrometer head at *L* is of prime interest, for with a good lead screw it would permit accurate setting of the depth of the teeth. Of even greater significance is the micrometer head at Z. Working through the 12″ index plate, with its circles divided for all the common numbers of teeth in clockwork from 49 to 365, the 60 divisions on this micrometer head provide for full differential indexing, over 150 years before this feature appeared on the milling machine. Hindley's method depended only on an accurate tangent screw[21] and on a head accurately divided in 60 units. Both of these were not too difficult to accomplish, and so this represents an advance on Fardoil. Hindley could then divide for *any* practical number of teeth with ease.

Hindley also provided for easy realignment of the center of the cutter with the center of the gear arbor after the cutter has been removed for resharpening. This was done by loosening screws *k* and *l;* then by means of the handle *i* the center of the cutter is brought into the center of the notched gage *P*. Note also that Hindley's engine cut flat bottoms on the spaces between the teeth, rather than a concave arc as in most previous engines. For wide gears it had been necessary to file these flat by hand. It was now done by the engine by means of the yoke and guides (shown in 2 and 3 of our Fig. 7) which give the cutter arbor a motion in a plane parallel to the axis of the gear and keep the axis of the cutter arbor at right angles to that of the gear arbor. This machine could also cut pinions and racks by means of a special attachment. For spacing the teeth on a rack a worm was provided with a click mechanism to give the precise distances between the teeth. (See Fig. 7, items 1-6.)

In 1783, with the wheel-cutting engine of Samuel Rehé, (?-1806?), we have a clock-maker's tool about to make the

21. It was in this connection that he developed the Hindley worm screw. For the first machine for cutting these commercially, see *Am. Mach.*, July 18, 1907, p. 85.

transition to a gear-cutting machine. And we find a number of other interesting features as well. Fig. 8 shows Rehé's machine as taking on the heavier, more massive frame—both in the base and in the upright—necessary for heavier gear cutting. Note the very solid box-type casting, and provision for smooth vertical working of the cutter arbor slide by means of springs and a counterweight. The diameter of the index plate has been increased to 19 inches, and to avoid constant counting a moving index is provided. The plate was divided from 37 to 720, but its micrometer feature is divided in only 30 units. The moving index does not keep the indexing arm always tangent to the dividing circle in use and its point at

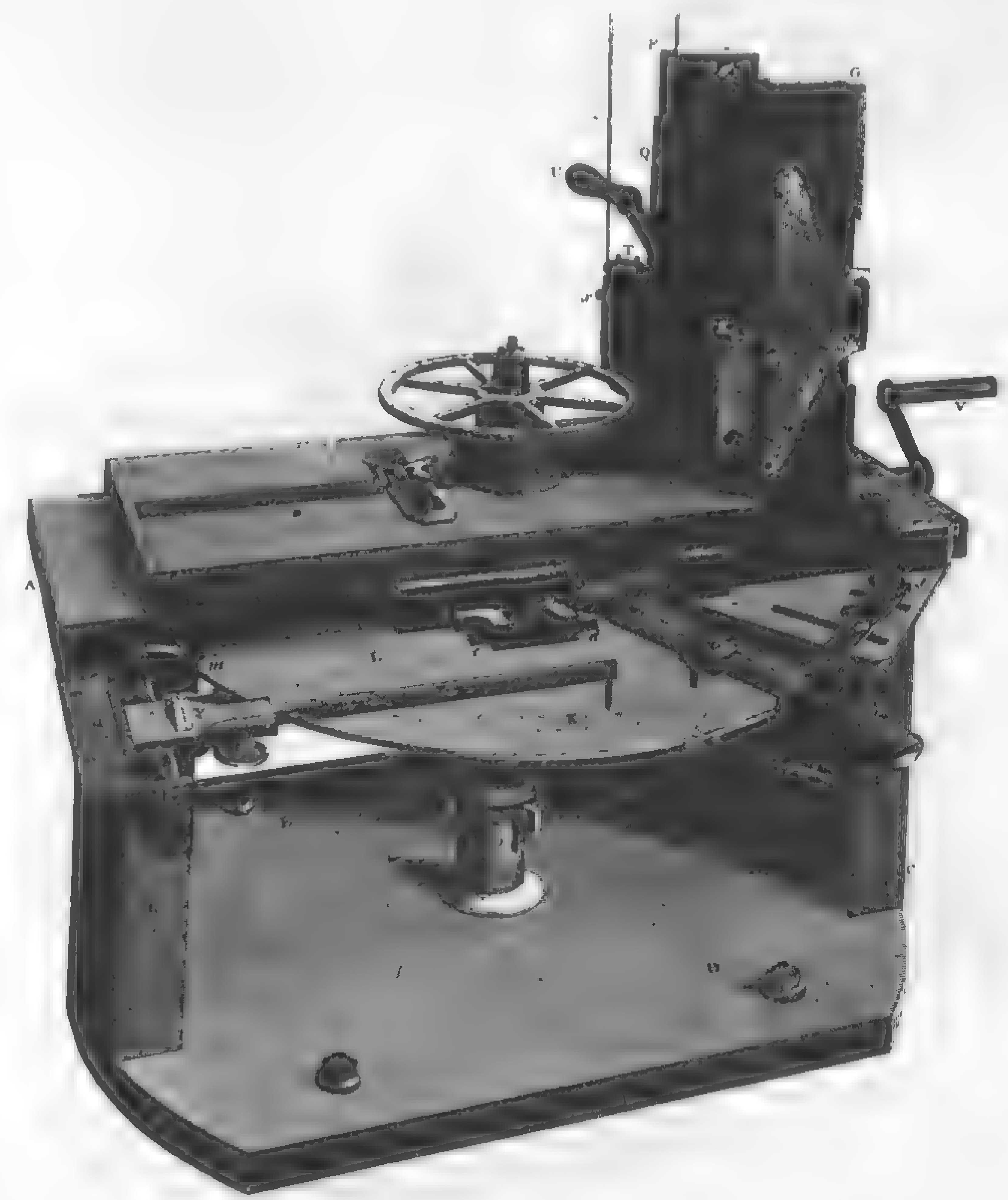
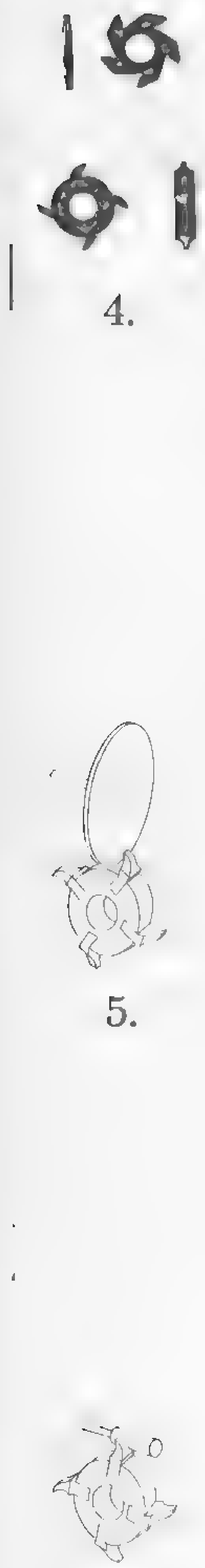


FIG. 8. REHÉ'S ENGINE, 1783 (*Rees*)

right angles to the central arbor of the plate. The indexing of this engine was therefore probably inferior to Hindley's even though simpler.

Rehé's machine, while simpler and more rugged than Hindley's, was probably less accurate and not much more convenient; e.g., why not put the handle *U* on the opposite side, so that the operator could reach all controls from one position?

But Rehé's machine has other, more important, features. One is his cutters. These have much coarser teeth; they are really milling cutters rather than rotary files. They have *formed* teeth, easily sharpened. The teeth of some are clearly inserted, as can be seen in Fig. 9 (his figures 2, 4, 5, and 6) which shows these cutters and Rehé's tool grinder. The form of these cutters is neither involute nor epicycloidal.



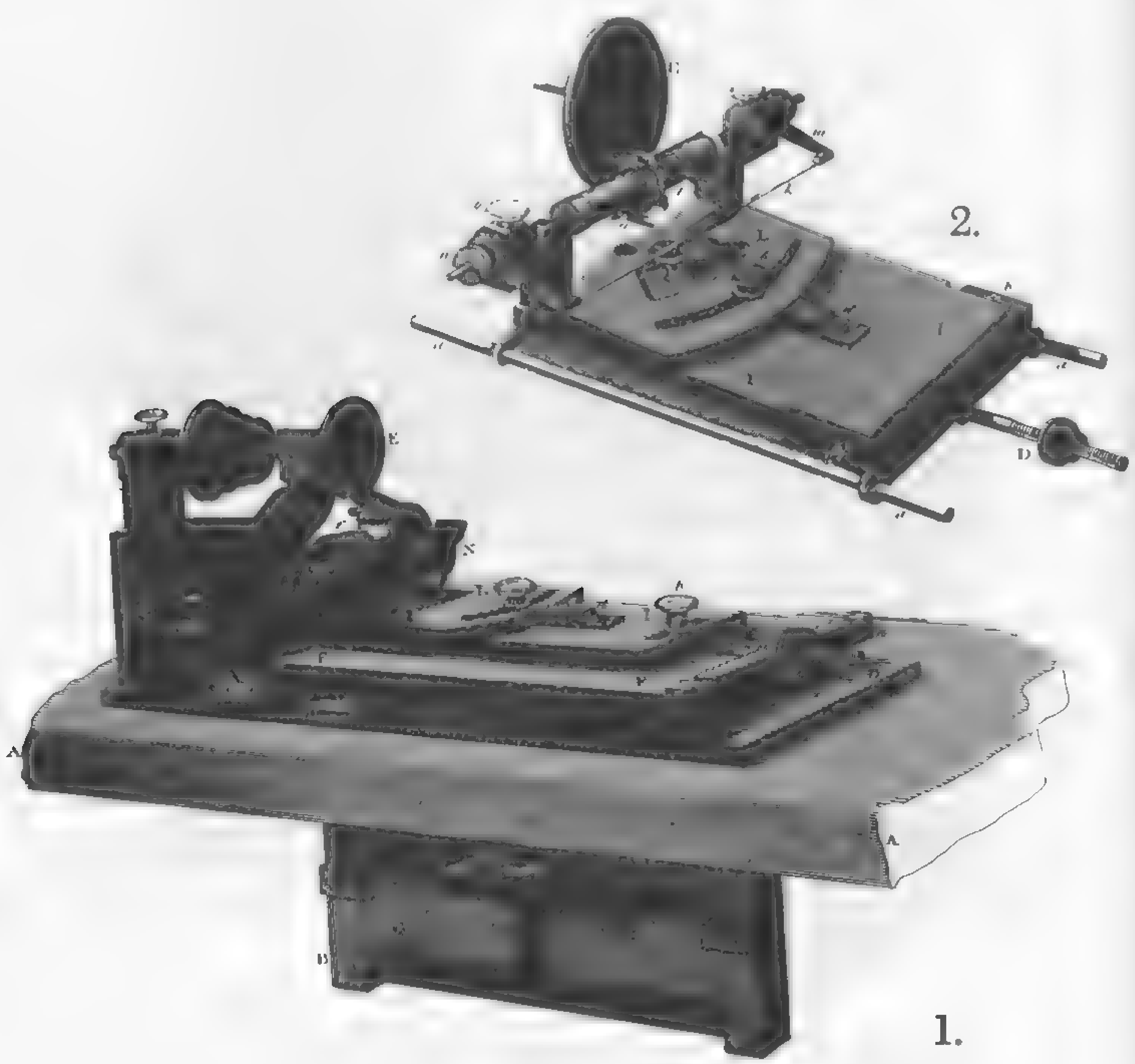


FIG. 9. REHÉ'S CUTTER GRINDER (*Rees*)

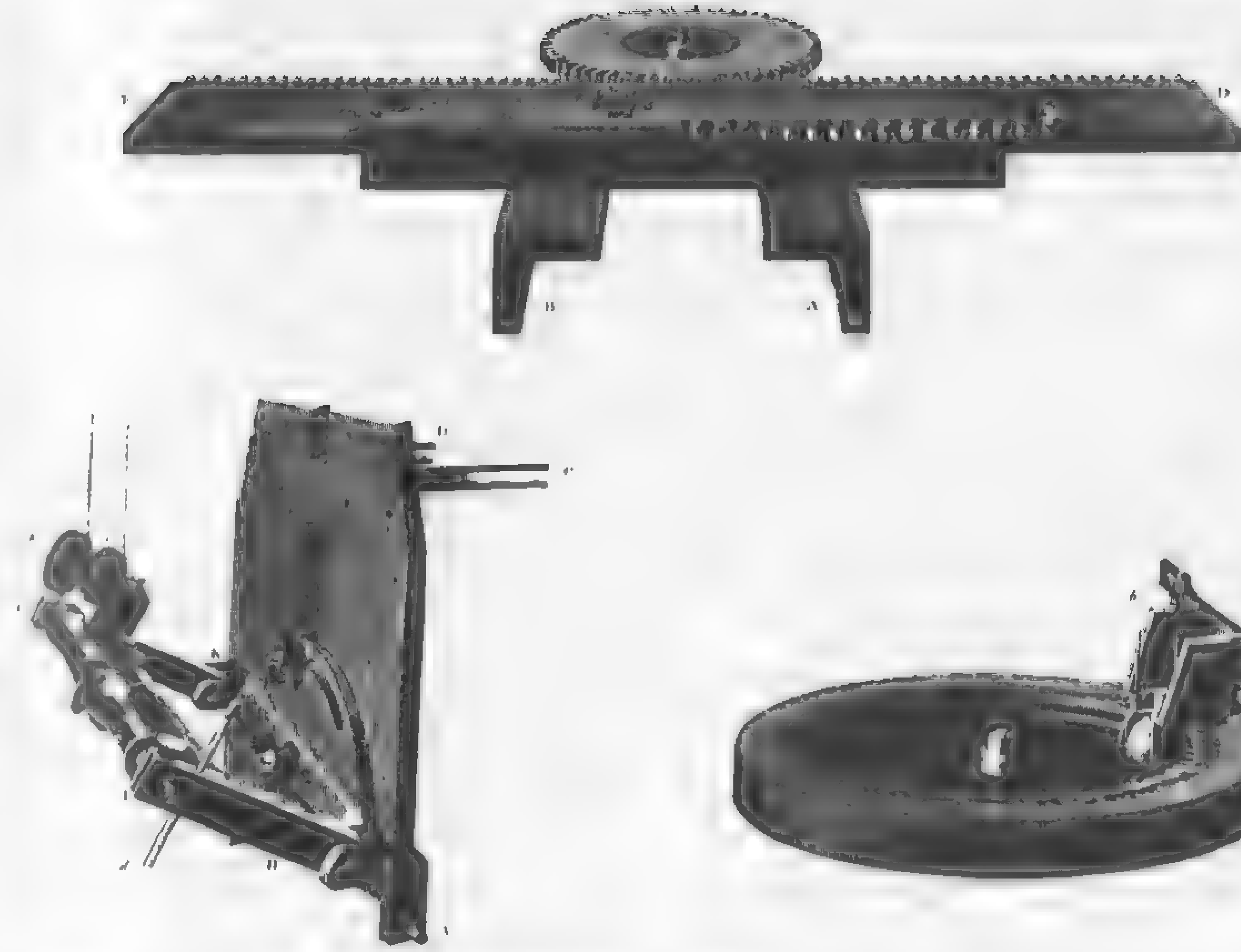


FIG. 10. REHÉ'S ATTACHMENTS FOR CUTTING INTERNAL GEARS AND RACKS (*Rees*)

Rehé's recognized the value of these mathematical tooth forms, as well as the need for cutters of various pitches, but he says that in practice all this is too great a refinement for the trouble involved. His dictum has been generally followed by clock makers ever since. Reid does tell us that these cutters "rounded the teeth at the same time as they cut the spaces."[22] They did, at any rate, produce better teeth than those rough machine cut and then hand finished. Rehe actually ground his cutters, using a mixture of emery oil, on a cylindrical grinder inclined at an angle to give an elliptical cutting edge as a practical approximation to the correct form. His sharpening engine permitted very precise grinding of the cutting edges and surfaces of his gear cutters.

Rehé also provided an attachment to cut a wheel which could engage a worm. It would cut, however, only a very crude approximation, and had been anticipated by Hulot's engine as described in Berthoud. A device for cutting a rack, much simpler than Hindley's, was also available. (Fig. 10)

22. Reid's article "Cutting Engines" in Rees *Encyclopedia*.

More significant is Rehé's attachment for cutting annular wheels (Fig. 10). Unfortunately Reid does not tell us if Rehé's recognized the need for a different cutter for this internal gear. Camus had not worked out the theory; Hawkins had.

For the clock makers Rehé's machine is not much of an advance, if any, over Hindley's. From our standpoint it marks a transition from a clock-maker's tool to a machinist's gear-cutting machine.[23] His cutters and his tool grinder represent a most important step, for Rehé had a clear picture of the formed-tooth milling-type cutter, at least for spur and annular gears, and a clear recognition of what the form should be, even if he discards it as not worthwhile for clock gears. Reid wishes that the clock makers would adopt more widely the machine-cut gears Rehé made possible, as they had not even by Hawkins' day. Whether Rehé knew how to grind his cutter teeth so as to give a correct epicycloidal or involute form we do not know, but it seems unlikely until Sang had developed the necessary theory for generation, although a templet could be used.

The Instrument Makers

The clock-maker's wheel-cutting engine had three basic problems: (a) careful alignment of the axis of the cutter with the axis of the gear blank, (b) the form of the cutter to give the correct shape to the teeth, and (c) the precise graduation of the index plate. With Hindley and Rehé the first two of these problems were solved, or at least an acceptable solution was found. Even if it was not sufficiently accurate for the gear-cutting machine, it could be made so. To the last of these problems, the accurate division of the index plate, a great deal of attention was given, even by the clock makers. Here they, and later the gear cutters, were able to depend on the precision work already done by the makers of "mathematical instruments." Both astronomy and surveying, as well as navigation, required highly accurate means of measurement of angles.

Leonardo da Vinci, Tycho Brahe, and many others had been concerned with what the 18th century called the "dividing engine."[24] As late as 1676 Flamsteed's large sextant (radius 6 feet 9¼ inches), constructed to the best standards of the day, and using Hooke's tangent-screw principle, had a backlash of 1 minute of arc. This is an error of 1/50 inch in the rack! The dividing had been done by Tompion by methods we do not know. The first attempt at accurate division, of which we know the actual methods used, is that of Rømer at about this same time. However, his results are of doubtful accuracy, since he used the stepping method but failed to check back. In 1725 Graham used the bisecting method, but did not take account of variations in temperature. In 1740 Hindley used the clever device of bending a brass strip into a circle.[25] But Bird's dividing engine of 1740 was the first to take full account of temperature variations and to use the vernier. By this means Bird could compare equal distances along the arc to within .001″ and estimate to .0003″.

The real foundations for precise division of the circle were, however, laid by Marie Joseph Louis, duc de Chaulnes (1741-1789) in 1768.[26] His method of using the microscope

23. Mr. K. R. Gilbert, of the Science Museum, South Kensington, kindly pointed out to the author that the wheels being cut in Rehé's machines shown in Figures 8 and 10 are clearly not clock wheels. It is quite possible that Rehé constructed these engines for cutting the wheels of the silk-reeling works built by Boulton about 1780 for the East India Company. See Watt's memoir of Boulton, dated Glasgow, 1809, printed in H. W. Dickinson *Matthew Boulton*, Cambridge University Press, 1937, Appendix I.

24. The full story of this development will be given in a later monograph on the History of Shop Precision of Measurement.

25. See his letter to Smeaton of Nov. 14, 1748, in *Trans. Am. Soc. Mech. Eng.*, 1893, p. 1233, and Smeaton's account in *Phil. Trans. Roy. Soc. Lond.*, 1786, Vol. LXXVI, p. 1.

26. See his two papers in *Mém. Acad. Roy. Paris*, 1768, "Nouvelle méthode pour diviser les Instruments de Mathématique et d'Astronomie," and "Description d'un Microscope et de Micromètres destinés à mésurer des partes circulaires ou droites avec la plus grande précision."

and comparing equal arcs later formed the basis of Ramsden's work. The elder of the two Troughtons, John, used an ingenious modification of Bird's method.

Ramsden's "coaxing method" combined Bird's and de Chaulnes' methods to give extremely accurate results. His first engine of 1768 was 30 inches in diameter. His second was 46 inches in diameter and is now in the collections of the United States National Museum, together with much of his equipment for making it.[27] Later Edward Troughton used a roller visual method which took much less time than either Bird or Ramsden, yet gave comparable results.[28]

We need not here concern ourselves with the later history of the dividing engine in the work of Troughton & Simms, Wurdeman, Rogers, and the Cornell engine. Suffice it to note that to supply the need of very precisely divided gears for the rotary press and for the Thorne typesetter, R. Hoe & Company went to some lengths to get a very accurate dividing engine for the manufacture of gears and other parts.[29] The Mundo Iron Works had a similar problem.[30] And even more accurate division was required for electrical purposes by the Westinghouse Company.[31]

Our immediate concern is only with the use of these precision dividing engines in early gear-cutting machines. The same Hindley who designed the wheel-cutting engine had also produced an early circular dividing engine. Rehé's machine was divided upon Ramsden's engine. And in 1833 we find Brown and Sharpe advertising, not only that they will supply "Spur, Spiral and Bevel Geer, and Screws for Worm Geer. Dividing Plates for all sized Engines graduated

27. See Ramsden, for detailed description of his method. The U. S. National Museum Catalogue Numbers are: for the Dividing Engine, 215518; for the device for cutting the worm, 215519.

28. Edward Troughton, "Method of Dividing Astronomical and Other Instruments," in *Phil. Trans. Roy. Soc. Lond.*, 1809, pp. 105-145.

29. *Amer. Mach.*, 1898, pp. 412, 419.

30. *Amer. Mach.*, 1898, p. 534.

31. Halsey, p. 51.

in the most perfect manner," but that "They have a Dividing Engine for making the most accurate graduations for Mathematical and Nautical Instruments. . ."[32]

In 1856 they had undertaken the construction of an improved circular dividing engine, which was completed in 1858. This engine was graduated from that of William Wurdeman, which stemmed from the Coast Survey's Troughton & Simms engine.[33]

Quite clearly, then, the work of the instrument makers gave the necessary precision of indexing to the early gear-cutting machines. The problem by, say 1830, was to fit all these parts together into a practical production gear-cutting machine. All the necessary elements were present.

32. Advertisement, *Providence Journal*, April 22, 1833.

33. Brown and Sharpe letter files, Book 7, pp. 127, 202; Book 9, p. 255; Book 13, p. 294.

III Production

Gear-Cutting Machines

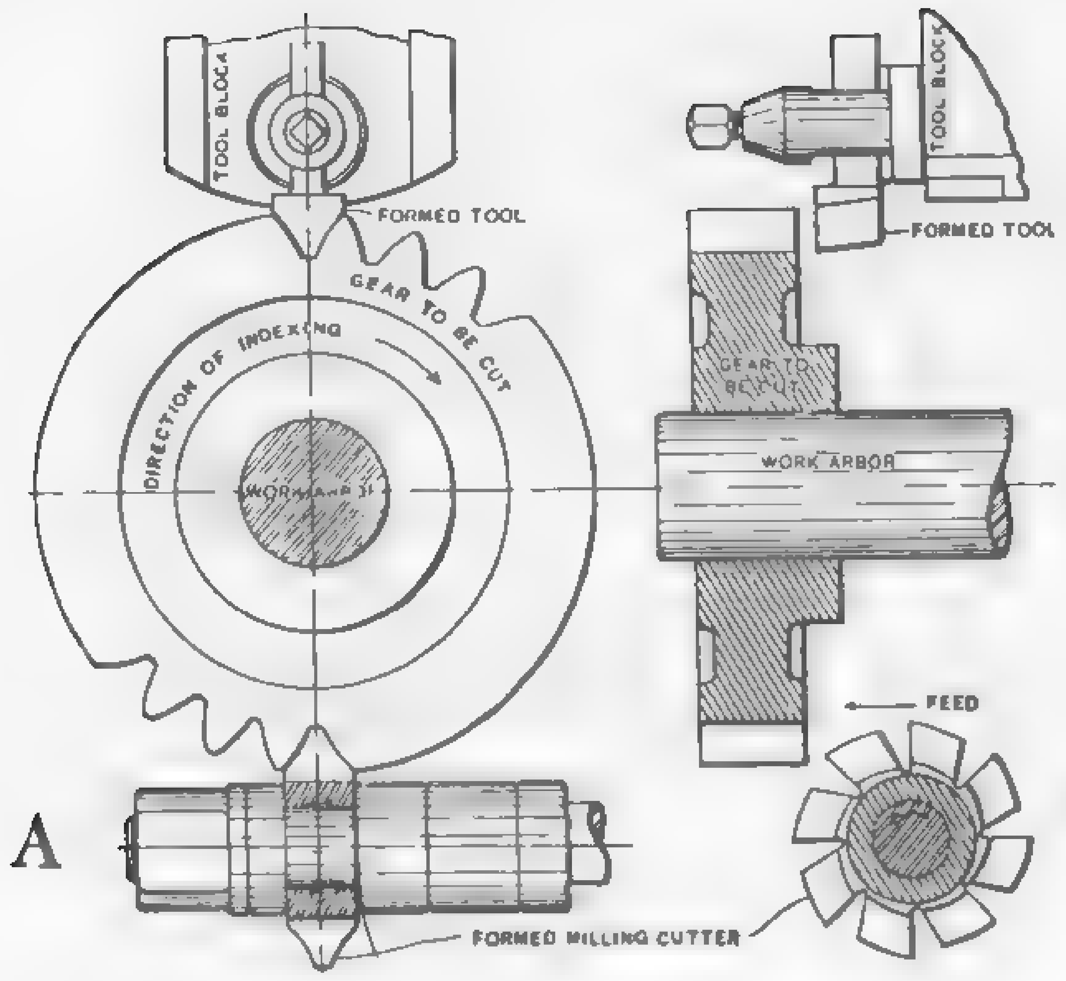
TYPES OF GEAR-CUTTING MACHINES

THE FIRST
PRODUCTION MACHINES 1800-1850

EVOLUTION OF THE
PRODUCTION MACHINE 1850-1910

AUTOMATIC GEAR-CUTTING
MACHINES, 1875-1910

PRECISION—GEAR GRINDING,
GEAR SHAVING, AND GEAR MEASUREMENT



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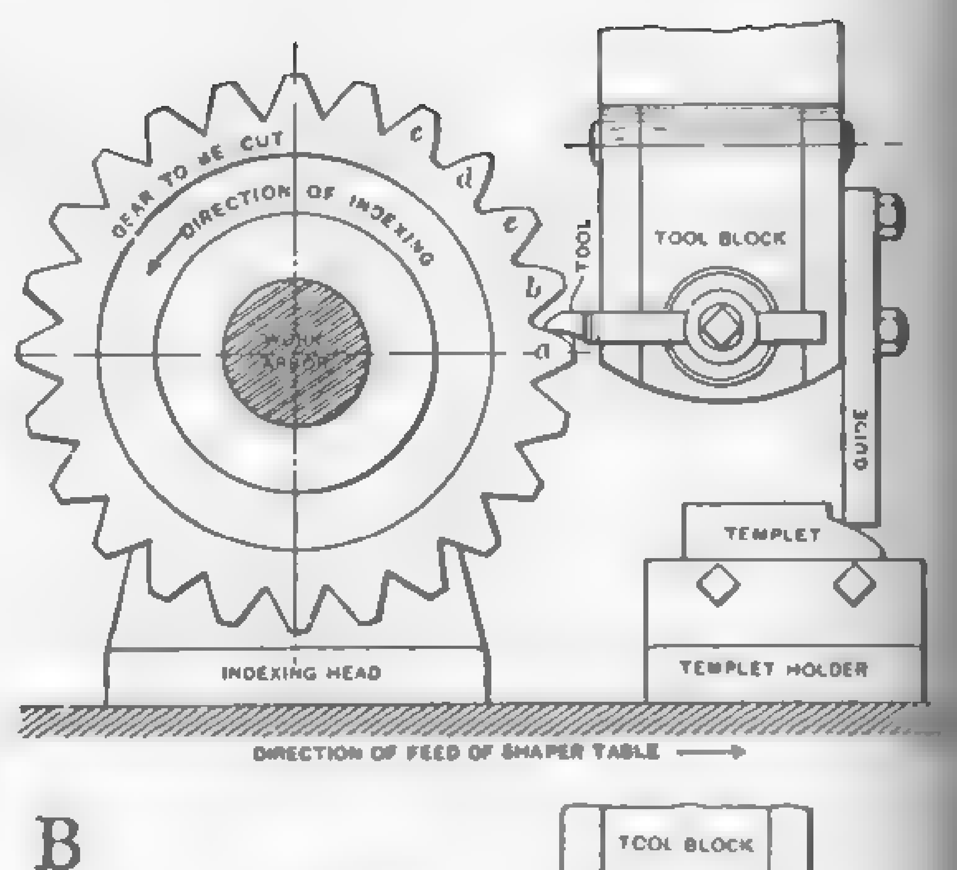

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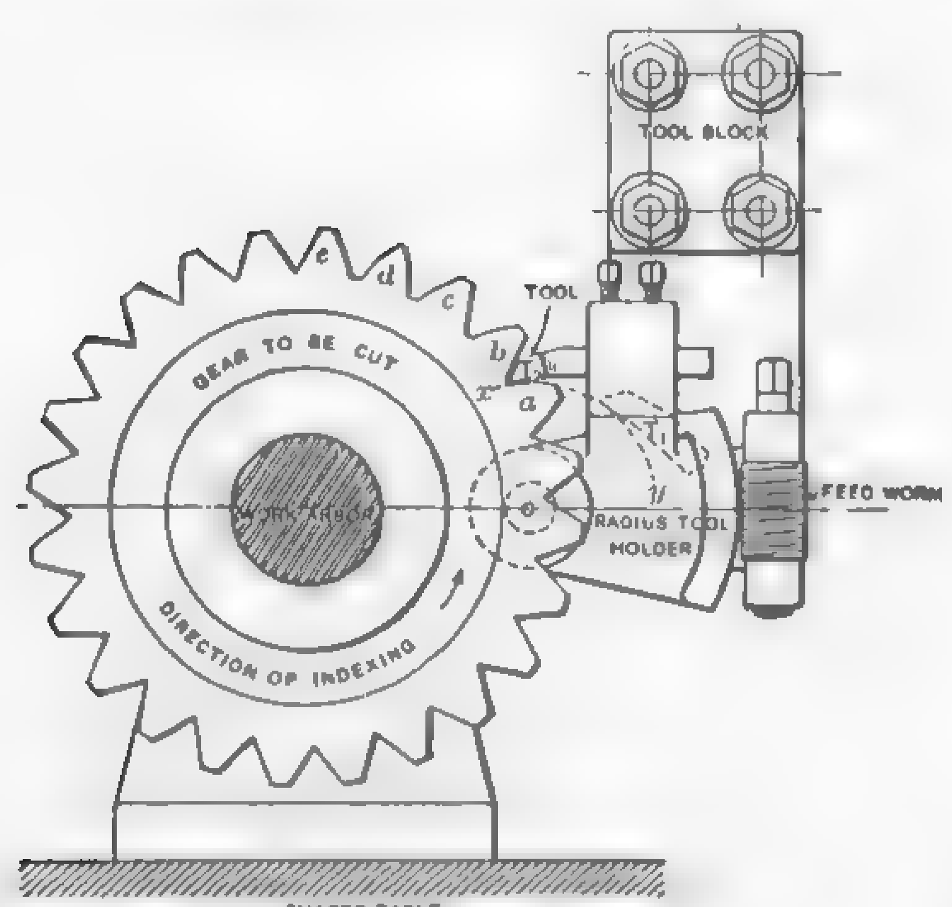

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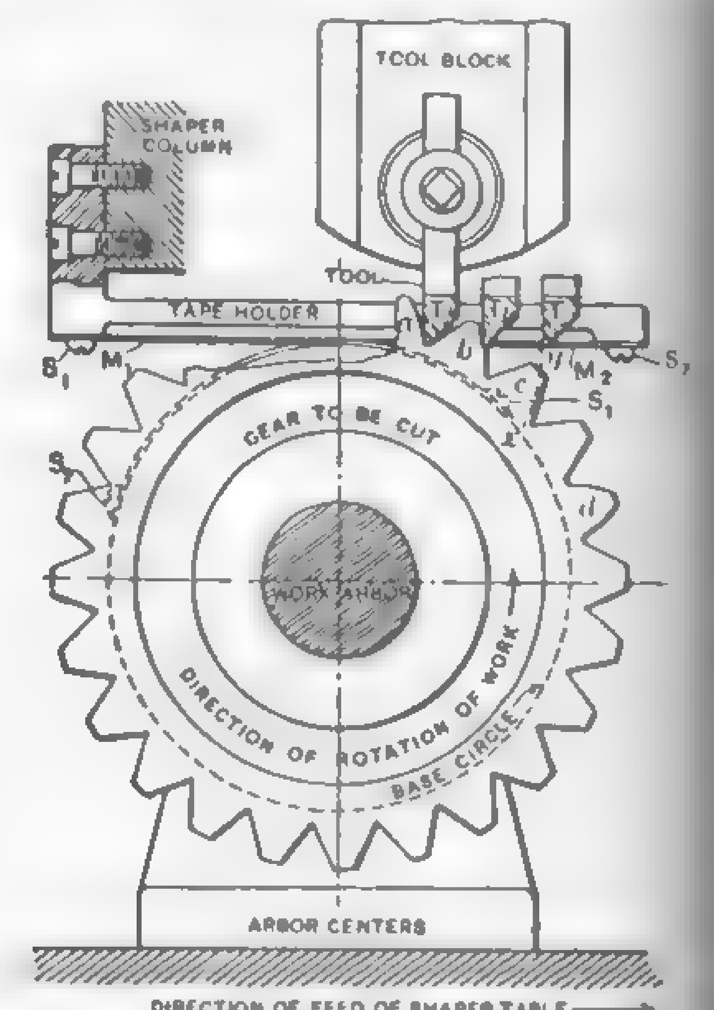

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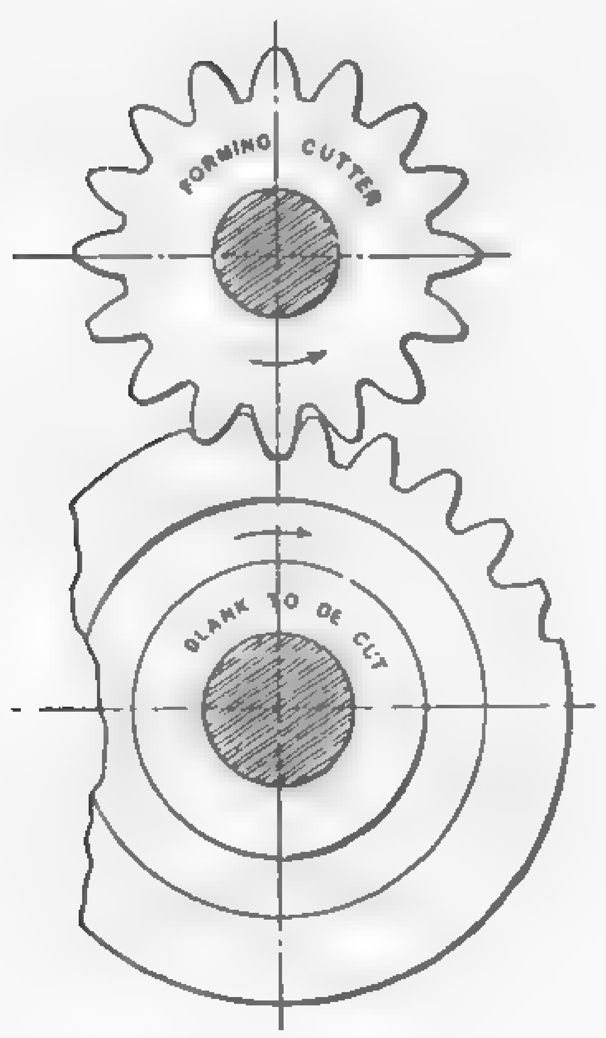


E

FIG. 11A. THE FORMED-TOOL PRINCIPLE: SINGLE-POINT AND MILLING CUTTER *(Flanders)*

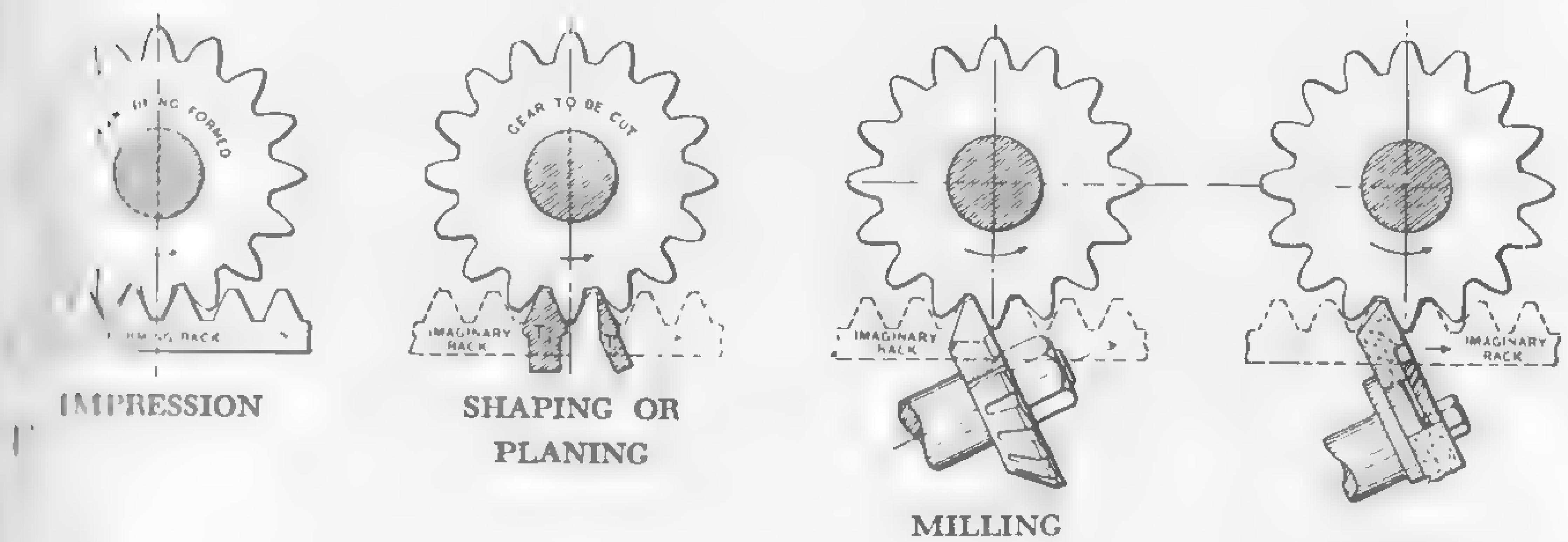
FIG. 11B. THE TEMPLET PRINCIPLE: AS APPLIED TO THE SHAPER *(Flanders)*

FIG. 11C. THE ODONTOGRAPHIC PRINCIPLE: TOOTH FORM OUTLINED BY MECHANICAL MEANS *(Flanders)*

FIG. 11D. THE DESCRIBING-GENERATING PRINCIPLE: POINT OF TOOL CONSTRAINED TO PROPER OUTLINE *(Flanders)*

FIG. 11E. THE MOLDING-GENERATING PRINCIPLE: TEETH SHAPED BY INTERMESHING CUTTER *(Flanders)*

FIG. 11F. THE FOUR METHODS OF OPERATION APPLIED TO MOLDING-GENERATING PRINCIPLE OF ACTION *(Flanders)*



IMPRESSION

SHAPING OR PLANING

MILLING

GRINDING

Types of Gear-Cutting Machines

Accurate indexing and mathematical studies on gear-tooth form advanced, of course, along with the actual means of cutting the gears of various types. The problem of correctly shaping the teeth of metallic gears mechanically has been met in four principal ways: First, by *impression* (Fig. 11f). That is, the gear blank (or the material used to form the mold for a cast gear) is forced up against a correctly formed gear or part of a gear and the correct shape impressed into the gear (or the mold for casting it). Second, by having a *formed cutter* (Fig. 11a), either single or multiple (broach, grinding wheel, or more commonly, a milling cutter), move relative to the blank so as to cut a groove of the same cross section as itself, thereby forming the adjacent sides of two teeth. Third, by causing a cutter (single-point or grinding-wheel, but usually milling cutters) to reproduce in the blank the shape of a previously constructed *templet* (Fig. 11b). Using a single pointed tool, this was, until the development of the generating methods, the only method of cutting theoretically correct bevel-gear teeth. Fourth, by causing the tool (single-point, milling, hobbing, or grinding) to *generate* (Figs. 11d and 11e) the tooth curves by its motion relative to the blank. This method was applied to spur, and spiral gears, but was especially useful for bevel gears.

The first three of these methods require that the desired tooth form be laid out on a large scale (either in a drafting room or on an odontograph engine[1]), the resulting templet reduced to the correct size (to give increased accuracy), and reproduced in the cutter by means of some sort of pantographic machine, usually of the milling type. The complexity of this operation, when accurately done, led to two important results: (a) the widespread use of Brown and Sharpe formed cutters,[2] the first to be put on the market, and with them the 14½° involute-tooth diametral-pitch gear standard; (b) the evolution of the generating types of gear-cutting machines, which avoided the problem and produced teeth more accurate than required by the accuracy of the rest of the machines in which these gears would be used.

In finding our way among the bewildering numbers of gear-cutting machines, it will be useful to adopt Flanders' classification:[3]

1) By *type of gear*—spur and rack, spiral,[4] bevel, and worm gearing.
2) By the *way the tool is held and guided* to produce the teeth—formed-tool, templet, odontographic, describing-generating, and molding-generating methods (Figs. 10a to 10e).
3) By the *nature of the operation*—impression, planing or shaping, milling or hobbing, and grinding (Fig. 10f).

1. See Oscar Beale's of 1876 as described in L. D. Burlingame, "Historical Notes on Gear Teeth," *Machinery*, March 1924, p. 530, and in *Am. Mach.*, June 27, 1912, p. 1025. Also Beale's unpublished typescript autobiography in Brown and Sharpe files. Brown and Sharpe have Beale's Odonton engine in storage. His original odontograph was presented to Columbia University about 1927 and has since been lost, but Brown and Sharpe have detailed photos and drawings in their files. For MacCord's epicycloidal engine and pantographic gear-cutter engine, see *American Machinist*, 1880, p. 1. His was a kind of modified lathe.
2. They were imported, together with machines using them, in great quantities into English shops until at least 1908. See Thomas Humpage, "The Evolution and Methods of Manufacture of Spur Gearing," in *Proc. Inst. Mech. Eng.*, 1908, p. 657.
3. Flanders, pp. 2-13.
4. The controversy over "spiral" versus "helical" is best resolved by admitting

Not all combinations of these classes have been used, and some have been far more important in practice than others. The formed-tool method, with a milling cutter, has been most widely used. The templet principle has been applied principally to cutting very large gears. There have been no production machines using the odontographic method for spur gears.[5] In practice the Swasey method,[6] adopted for a while in the late 1880's by Pratt & Whitney, used the describing-generating method, but it was applied only to making gear-tooth cutters, not to cutting teeth in gears.

Of the formed-tool machines the planing and the milling methods have been most common, although hardened gears produced by a formed grinding wheel have in recent years become important.[7]

The molding-generating process has been widely used. The Fellows gear shaper has dominated the field of machines combining the molding-generating principle with the planing or shaping operation. But a number of molding-generating machines using a milling operation (hobbing gearcutters) have been developed, despite much discussion of the merits of this type. A molding-generating machine using the grinding operation has been used for hardened gears for automobiles.[7]

The First Production Machines, 1800-1855

During the early period of the production gear-cutting machine, (as distinguished from the clock-maker's wheel-cutting engine), there appeared not only many new types of gear-cutting machines, but also, in embryo at least, most of the types that were to follow.

that these gears are helical, but by also recognizing that they are almost universally referrred to as "spiral" and that the term will probably persist.
5. It has been applied to two types of bevel-gear machines.
See Flanders, pp. 267-274.
6. See Ambrose Swasey, *Trans. Am. Soc. Mech. Eng.*, 1891, pp. 265-274.
7. In both cases the problem has been to retain the correct form on the grinding wheel. Of course, neither type cuts the teeth from a blank; they only finish them. See Flanders, pp. 65-66, 99-103, for early examples of this type.

The clock makers, as we have seen, had used almost exclusively the formed cutter—at first a rotary file, later a milling cutter. Little attempt was made to get a theoretically correct form; their wheel-cutting engines made only a rough cut which was usually finished by hand. To be sure, their "deepening engine" did do part of the finishing mechanically by the impression method, but these machines were not in common use.

As cast gears came more and more into use in the last half of the 18th century there were two trends: toward increased accuracy of gears (an evolution we have already examined), and toward increased sizes of gears. The clock-maker's wheel-cutting engine was not at all capable of taking work of production size and accuracy; not even Rehé's engine could produce a gear of *engineering* size.

Gear-cutting machines which seem to have evolved directly out of those of the clock makers, yet are clearly production machines, are those of James White (Fig. 12). In fact they anticipate much that was to come later. They date certainly from prior to 1824. While those that White tells us he had actually constructed are of clock-maker dimensions, he points out that they could be built to any desired size. White provides a number of features which would be most useful in larger sizes, *e.g.*, his method of accurately centering the axes of the dividing plate and of the gear-blank shaft by means of bushed bearings. He suggests using either the contracting brass bearings of the *instrument makers* or type-metal bearings. His machines are intended to cut his patented spiral gears—one for spur and one for bevel. By means of an ingenious yet simple arrangement of cams, levers, cords, and weights the gear blank is rotated to give the spiral. The cutter remains fixed; only the gear blank moves. White gives full directions on how to compute the form of the cam required to produce the gear desired.

In his spiral spur-gear machine[8] the cutter is of the milling type set at the necessary angle of the teeth. It is

8. White, *New Century of Inventions, ca.* 1824, pp. 120-130, and Plates 15 and 16.

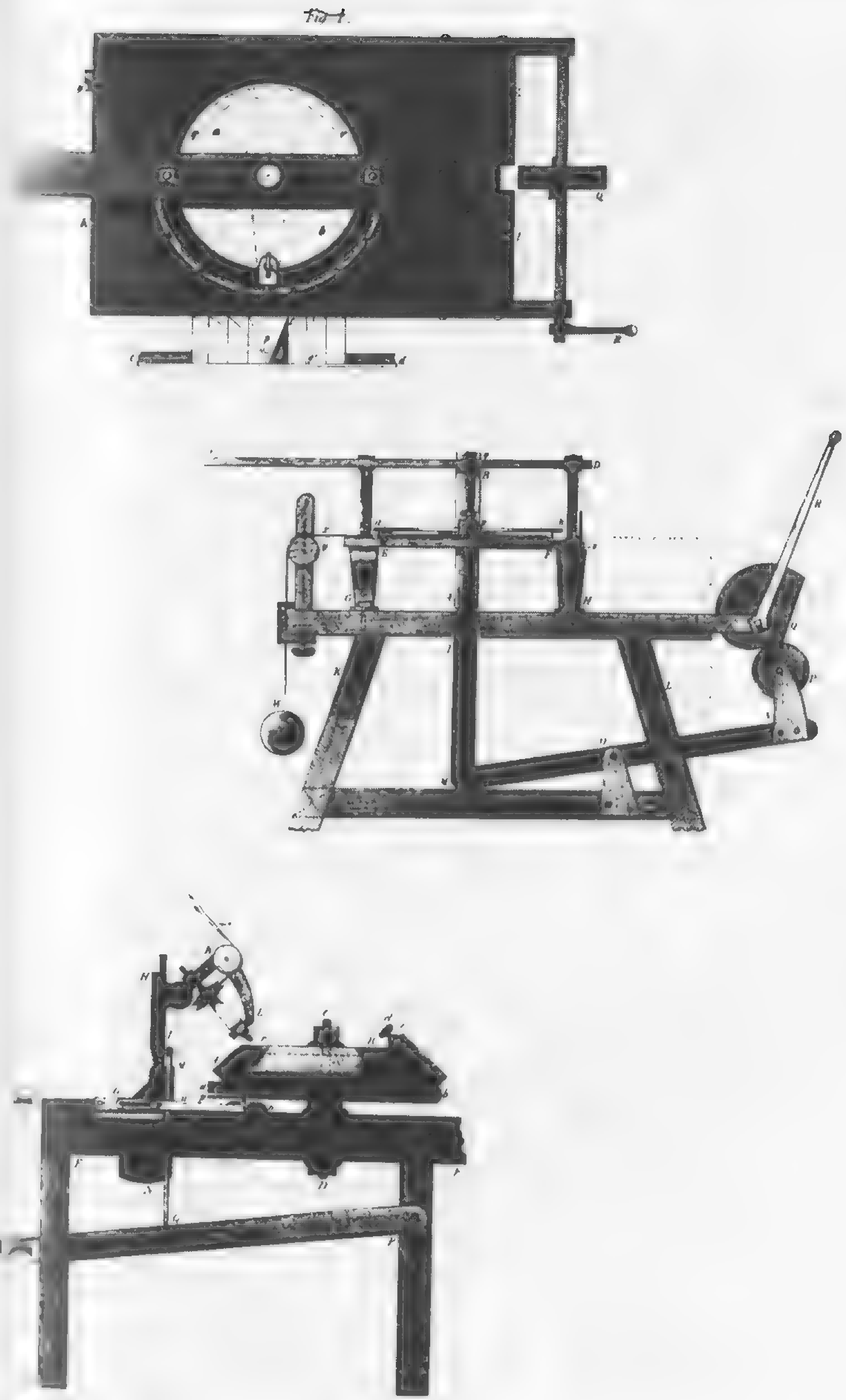


FIG. 12. JAMES WHITE'S ENGINES OF ABOUT 1820 (*White*)

driven by a cord-and-pulley arrangement, which would indicate that we do not yet have a full production machine.

In White's machines for cutting spiral bevel gears[9] the cutter is a slightly tapered end-mill. For the bevel-gear machines White also shows us how to compute the necessary cams.

White's gear-cutting engine is, then, a further transition from the clock maker to the engineer, but its early use to cut spiral gears, both spur and bevel, should be noted. It was the first to be able to cut gears of these types. Although it could be adapted to engineering work, it was not so used in production. One more step was necessary.

That step was taken prior to 1833 by James Fox, of Derby, who had in operation a gear-cutting machine clearly on the production level.[10] Fox had mounted on a very substantial, lathe-like frame a vertical spindle for the gear blank. Rather than the clock-makers' index disc, he used an index cylinder with a micrometer division by means of a screw. Fox even took care to have a conical pin enter the holes on the indexing cylinder, for accuracy under wear. The rotary cutter was carried on a rugged slide working in a sturdy upright guide by a rack and pinion. Hand feed by means of a handle and lead screw was provided. In his Plate IV, Wedding shows very clearly Fox's formed-tooth multiple cutters with inserted teeth. Although Fox's machine was apparently used only for making wooden patterns for cast gears, with a different type of cutter it would certainly have been adequate for cutting metallic gears of substantial size.

Here, as in so many other places in the history of machine tools, we find as a most significant figure that amazing Swiss, John George Bodmer (1786-1864). In his patent of 1839 he describes his gear-cutting machine.[11] His is of the

9. The gear cutter is White pp. 183-192, and plate 22. The machine for cutting wooden patterns for large gears is pp. 263-270 and plate 32.

10. Wedding, "Beschreibung einer von Fox in Derby erbauten Maschine zum Theilen und Schneiden von Radmodellen," in *Verhandlungen des Vereins zur Beförderung des Gewerbfleisses in Preussen,* 1833, pp. 37-41, and Plates I-IV.

11. Bodmer's British Patent No. 8070 of 29 May 1839, pp. 2-8, and sheet #1 of drawings.

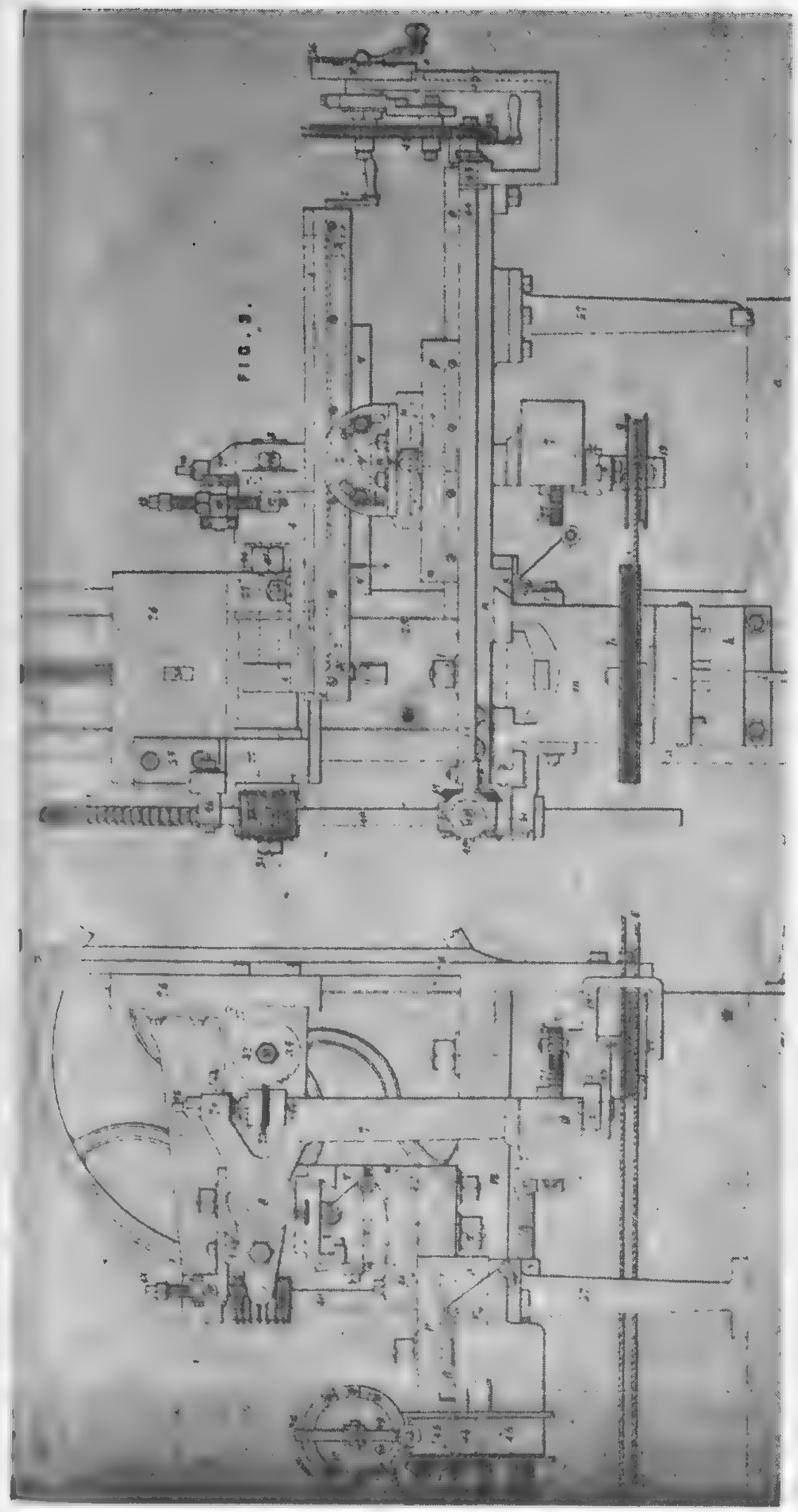


FIG. 13. BODMER'S ENGINE FROM HIS PATENT, 1839
(*H.M. Patent Office*)

formed milling-cutter type, and he gives a very full disclosure in his patent (Fig. 13). The machine was designed to cut spur gears, but Bodmer provided attachments for internal gears (both spur and some bevel), for worm wheels and for racks. In Sheet No. 1 his Fig. 9 shows an attachment "for giving a perfect shape to the teeth of bevel wheels, as patterns or for use."

Bodmer's cutters are of interest. Those for cutting wooden patterns for cast gears are of the fly-cutter type. Those for cutting actual metallic gears are of the milling type (Fig. 14), segmented for convenience in hardening. All are formed cutters. The problem of sharpening them gave great trouble and expense.[12]

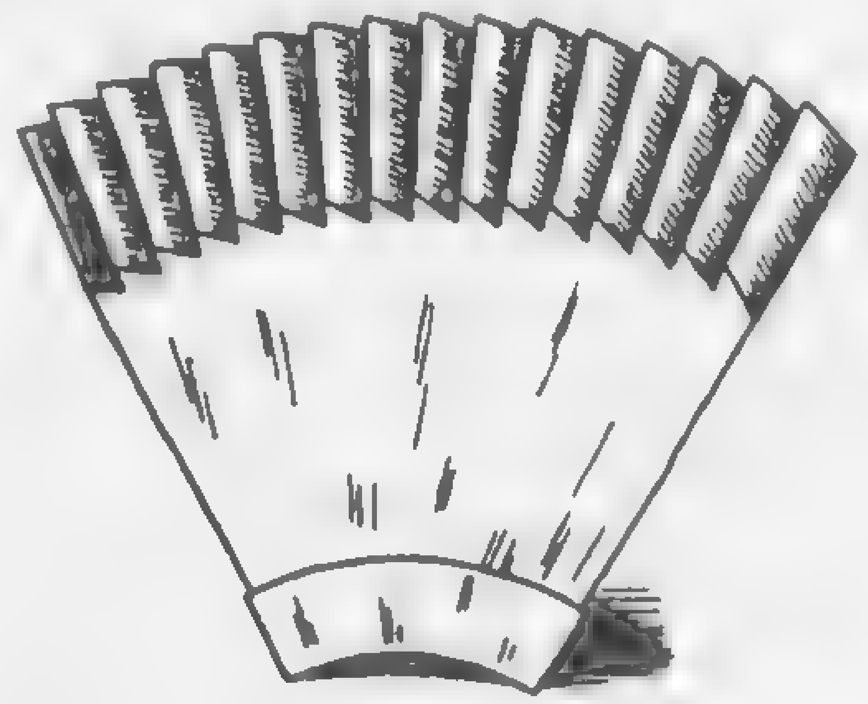


FIG. 14. BODMER'S SECTIONAL GEAR CUTTER (*Inst. Mech. Eng.*)

For "truing, cutting and shaping cutters for cutting teeth in wheels of metal and wood" Bodmer provides a "Pentograph."[13] This is itself essentially a gear-cutting machine using the templet method (reduced by the pentograph) and using a milling cutter. Bodmer also includes a cutter-grinding attachment for shaping the cutter by use of a copper form, oil, and emery.

Just when Bodmer began cutting gears, some of which we still have, is difficult to say—probably between 1820 and 1830. A wheel-cutting engine of 1824-1834 is shown in Fig. 15. This is probably the work of Richard Roberts, though it

12. See R. S. Woodbury, *History of the Milling Machine.*
13. Patent, pp. 8-11, and sheet #2 of drawings.

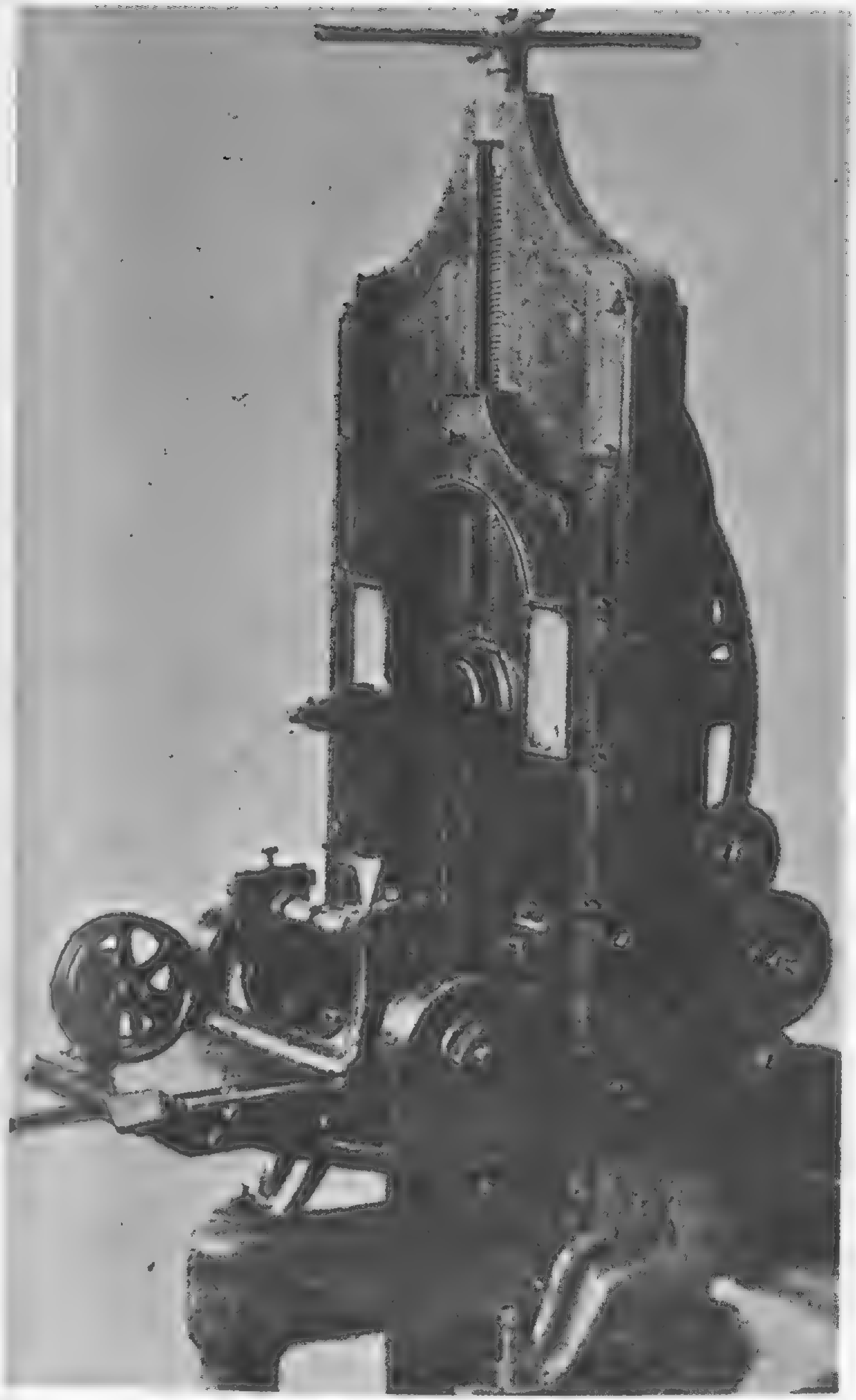


FIG. 15. ROBERTS' GEAR-CUTTING MACHINE, 1824-1834 (*Inst. Mech. Eng.*)

may be Bodmer's. The clock-maker's engine is here upended, for the dividing wheel and worm are behind the upright frame. It should be noted that a set of change gears is provided to give divisions for the number of teeth required. The gear blank is brought into position by a vertical screw, and the cutter is fed in horizontally by hand. By tilting the cutter head this machine can cut bevel gears. The method of drive of the cutter, probably a single-point formed fly cutter, is only a cord and pulley, which is still on the clock-maker level, but the rest of the machine is definitely an engineering design. This machine was still in use in 1908 in Salford, England. Bodmer later built at Bolton a machine, still in use in 1908, much larger and heavier and with a work table 6 feet in diameter indexed by a worm wheel 5 feet, 4 inches, in diameter and having 360 teeth of 0.9" pitch. In this machine the drive and support of the cutter were much more substantial. The design of this machine shows definitely the influence of Bodmer's patent of 1839.

In Buchanan's *Millwork*[14] are shown two gear-cutting machines by F. Lewis of Manchester (Fig. 16). Here we

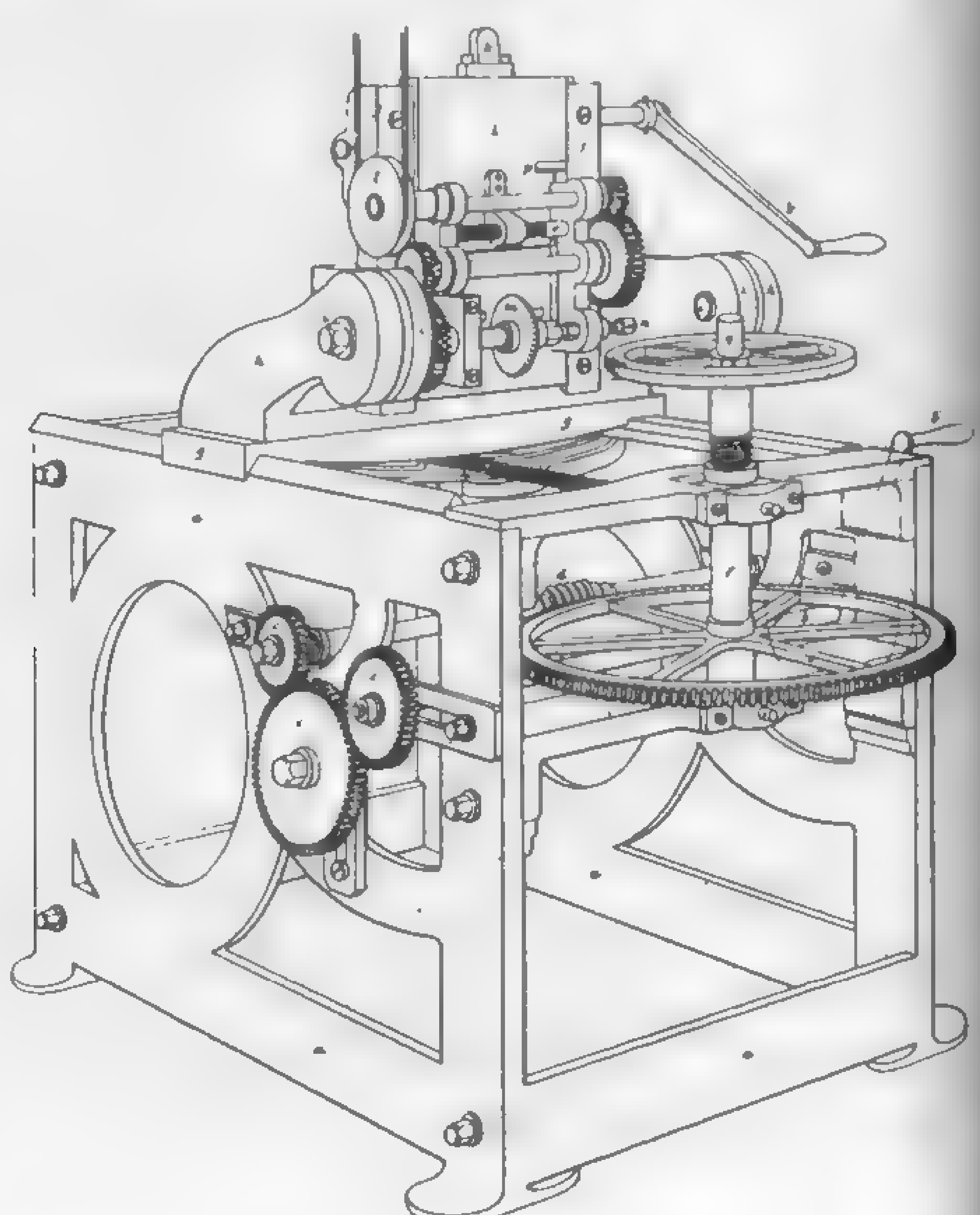


FIG. 16. LEWIS' GEAR-CUTTING MACHINE (*Buchanan*)

14. *Op. cit.*, pp. 440-447, and Plates 40, 40A.

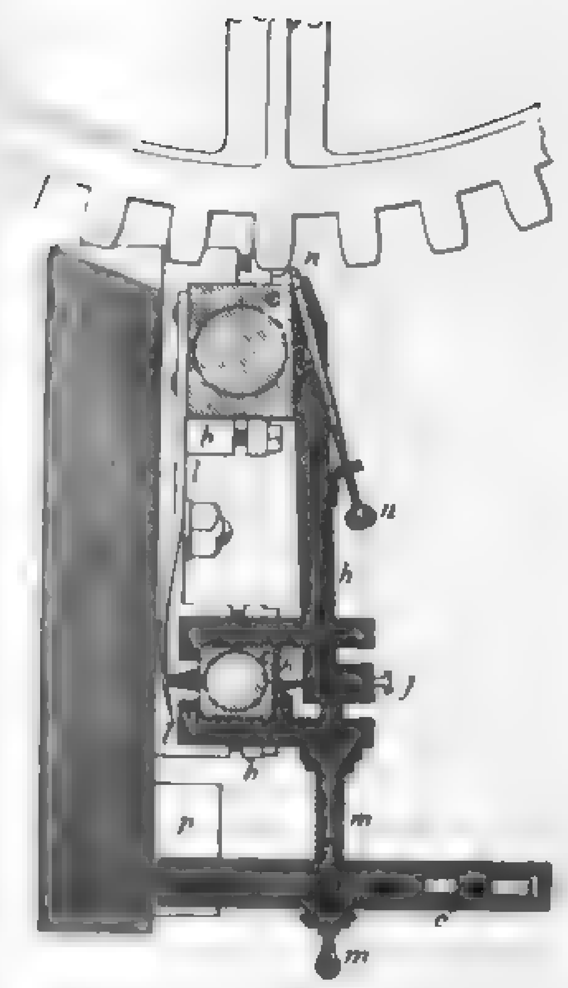


FIG. 17. GLAVET'S TEMPLET GEAR CUTTER, 1829 (*Buchanan*)

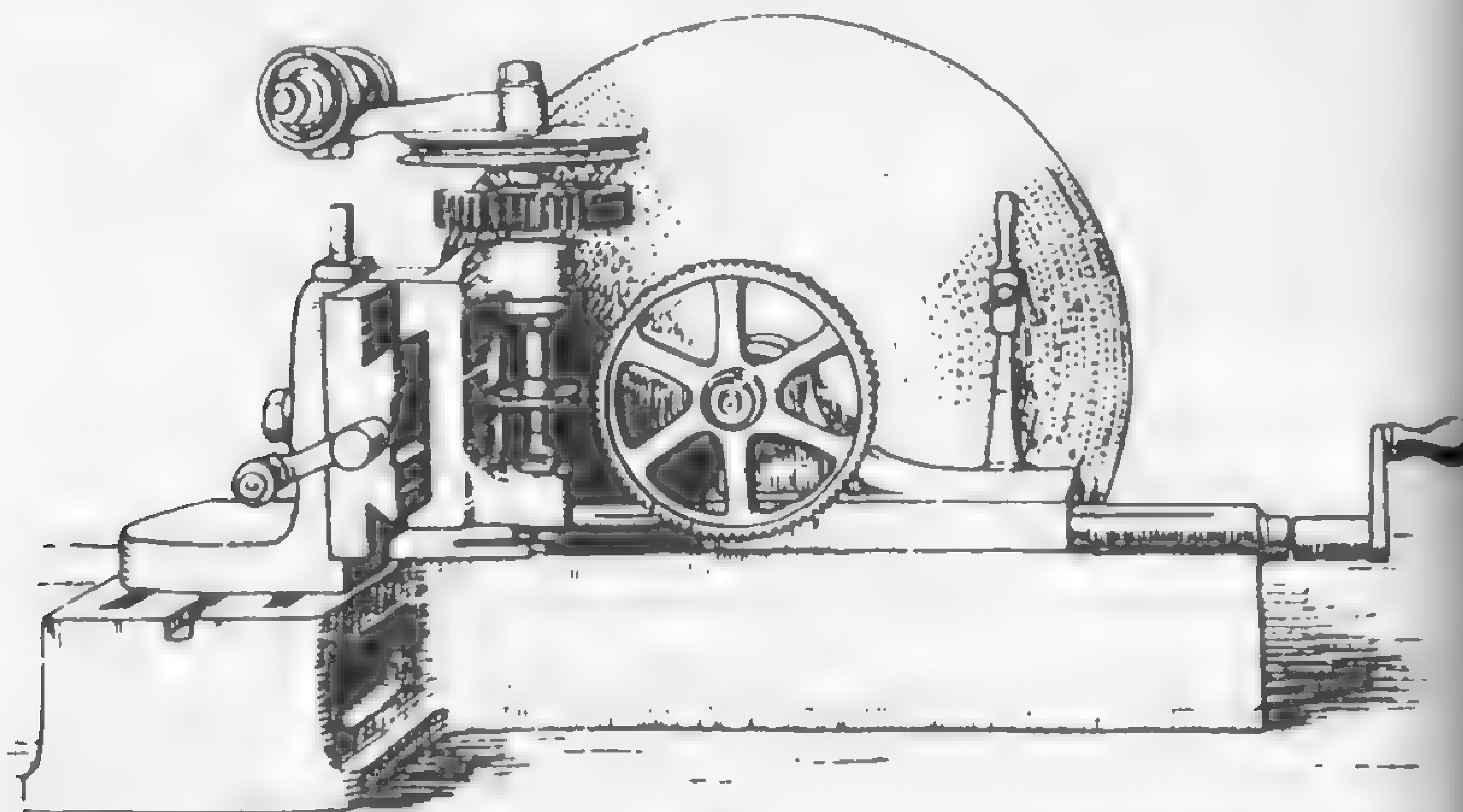
have machines definitely on the engineering level, even though their cutters are still hand powered. Both machines are capable of cutting spur and bevel gears and skew-bevel and worm wheels. The larger machine could cut gears 5 feet in diameter in iron, and 10 feet in wood, a width of up to 14 inches, and any pitch or number of teeth, by means of interchangeable gearing in the worm index.

Buchanan also shows a most interesting machine designed by Glavet.[15] This provides for a reciprocating shaper-like motion of a single-point tool controlled by a *templet* shown at *k* in Fig. 17.

Buchanan also illustrates a device whose invention has been doubtfully attributed to Maudslay and to Nasmyth, a modification of the screw-cutting lathe.[16] An indexing plate is secured to the driving pulley and an index arm attached to the bed. A vertical spindle carrying a pulley and a milling-type cutter are mounted on a shaft vertically on the tool rest. The cutter is driven by a cord-and-pulley arrangement and fed across the edge of the gear blank mounted between the lathe's live center and a sort of chuck on the spindle. This device would be of doubtful merit as shown; its indexing would be inaccurate and the drive would permit only small pitches to be cut. Certainly it is of little significance

15. Buchanan, pp. 448-448, and Plate 41A. See his French Patent of 6 Apr. 1829.
16. Buchanan, p. 409.

With the machines of Joseph Whitworth (1803-1887) of 1834-44 we have the first machines[17] with involute cutters, geared indexing and the cutter power-driven by a flat belt and pulley through a worm and wheel (Fig. 18a). By 1851 machines of this type were being made with self-acting feed.



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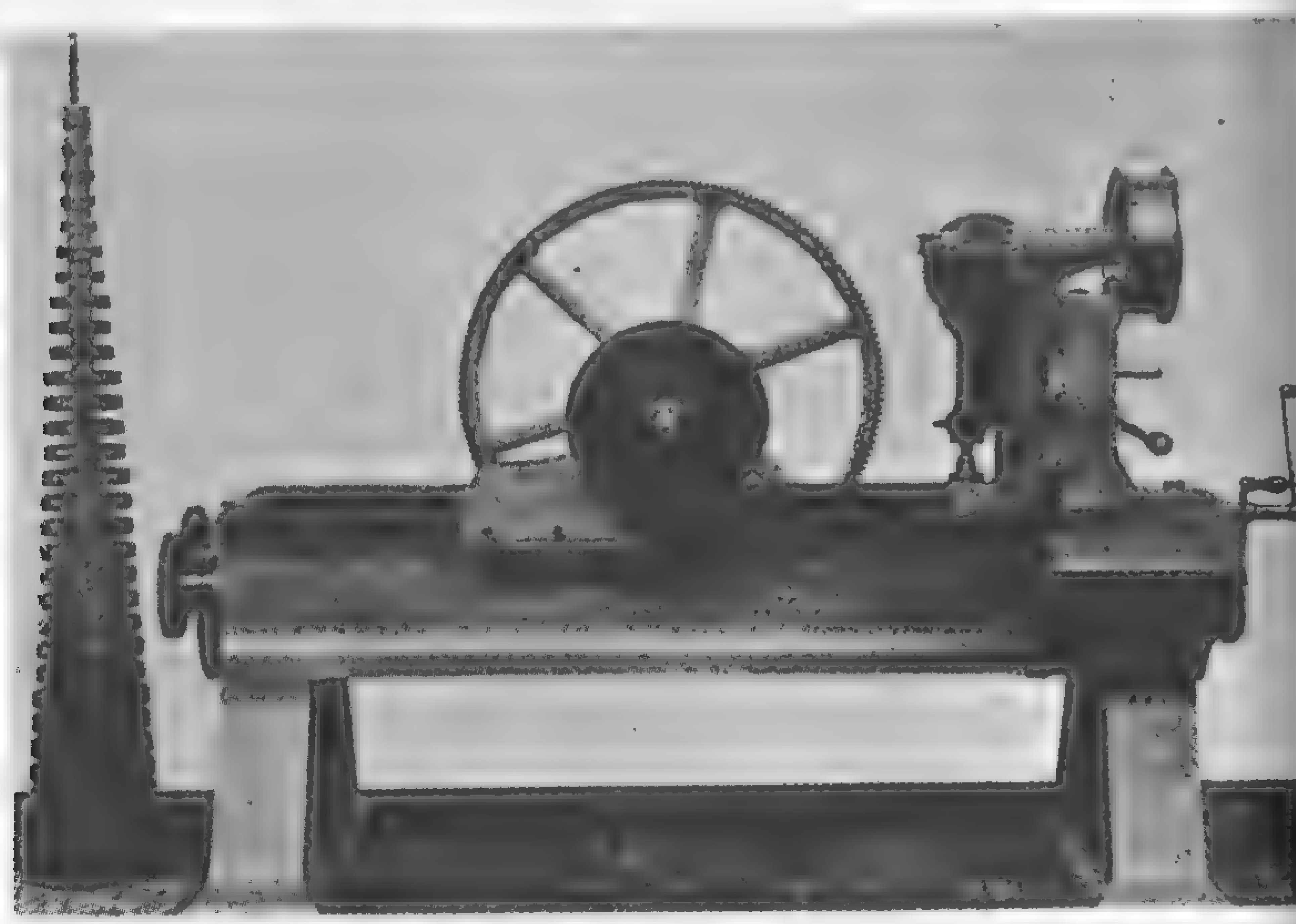


B

FIG. 18. WHITWORTH'S FORMED MULTIPLE-CUTTER MACHINES
(Inst. Mech. Eng.)

17. Humpage, p. 657, and plates 22, 23.

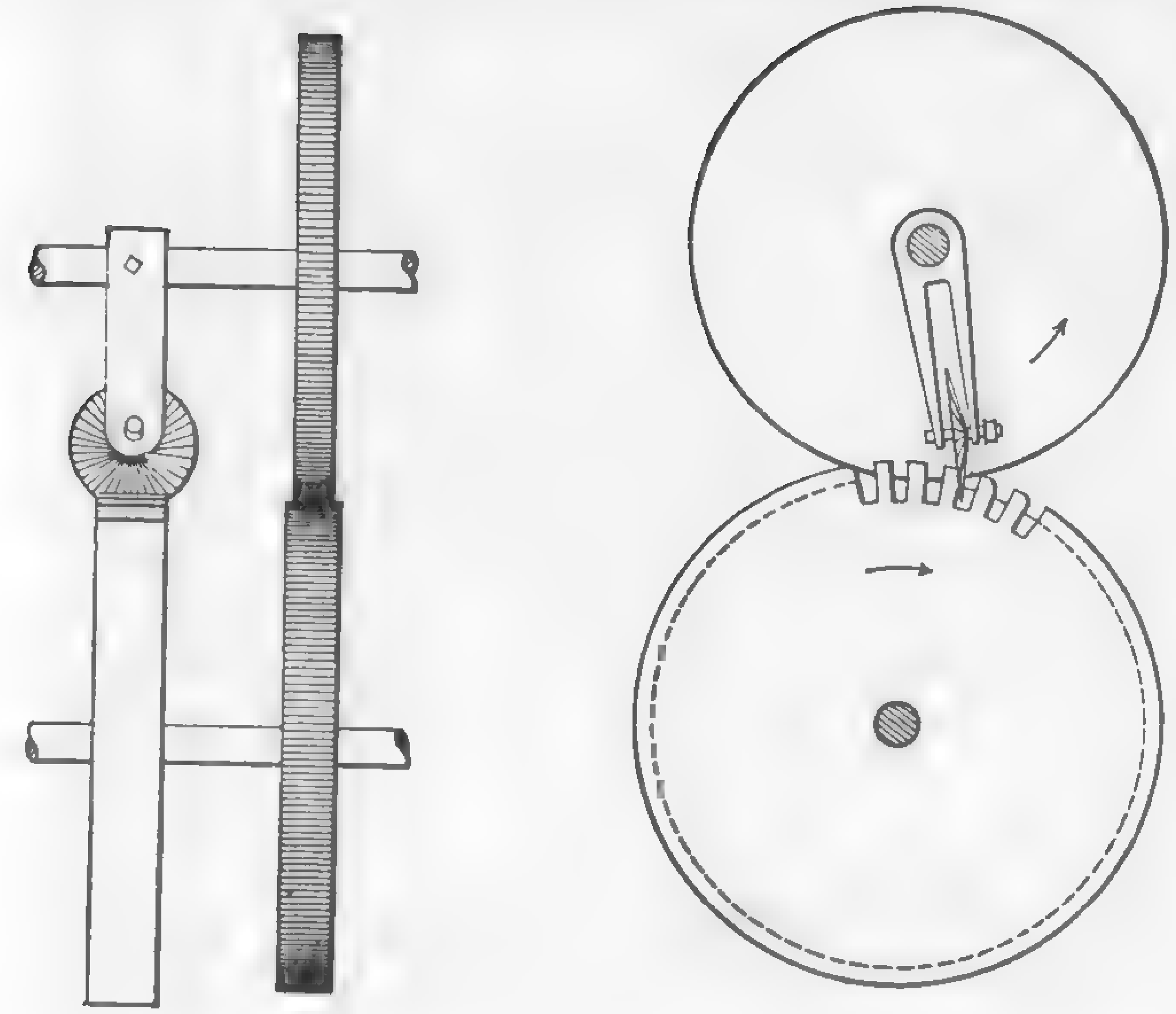


FIG. 19. SAXTON'S GEAR-GENERATING MACHINE, BEFORE 1842
(American Machinist)

Not only were the formed-cutter method (both single and multiple point) and the templet method already used in this early period, but we find the generating method in use as well. The first generating machine we know of is that of Joseph Saxton (Fig. 19).

"Mr. Saxton, of Philadelphia, now in London, who is justly celebrated for his excessively acute feeling of the nature and value of accuracy in mechanism, and who is reported not to be excelled by man in Europe or America for exquisite nicety of workmanship, made in Philadelphia an instrument for cutting the teeth of watch wheels, truly epicycloidal: or, rather, for curving them after they were cut down in the ordinary manner with radial faces.

"The wheel to be rounded being put on a vertical arbor, another arbor stands parallel to the first, carrying on a third but horizontal arbor a steel wheel file cut on the plane side, which plane side lies in a vertical plane passing through the axis of its vertical arbor. On the arbor of the wheel to be rounded is a circular plate equal in diameter to the primitive circle of that wheel. The edge of the plate is milled into teeth as fine as possible. This plate forms the base of the

epicycloid. On the other vertical arbor is a similar plate, but equal in diameter to the radius of the primitive circle of the wheel to be engaged with that about to be rounded. This plate is the generating circle.

"In working this instrument the flat-sided cutter is brought in contact with the side of the tooth to be rounded, the axis of the two vertical arbors, the face of the cutter and the line of the tooth all lying in one vertical plane. The cutter being set in motion by a band, the generating circle is rolled around the base, and thus one side of the tooth is rounded in a truly epicycloidal curve of the required dimensions.

"Upon this plan epicycloidal teeth of any magnitude might be cut with great expedition."[18]

To this full description need only be added that since the describing circle is equal in diameter to the *radius* of the meshing gear, the resulting epicycloidal faces will mesh correctly with the radial flanks of the other gear.

A few years later F. Bashworth described another type of generating machine.[19] As shown in Fig. 20, this machine has a small rotary cutter *E* equal in diameter to the pins of the pin gear. The friction wheels *A* and *B* have diameters equal to those of the pin gear and the gear being shaped respectively. Bashworth provided bands *C* to prevent slip between the friction wheels.

We do not know if Bashworth's machine was ever constructed, and Saxton's clearly belongs to the clock makers. But they do establish the use of the generating principle at least as early as 1840.

The molding-generating principle had already appeared in Whitworth's patent of 1835 for a worm-wheel hobbing machine[20] in which the gear blank was geared to the hob-

18. It is thus described in the 1842 edition of Hawkins' translation of Camus, but without any illustration. Fig. 19 is the reconstruction by Howard A. Coombs in *Am. Mach.*, July 23, 1903, p. 1043. Saxton evidently returned to the U.S.A. later, for in 1855 he was in charge of the Troughton & Simms dividing engine at the Coast Survey in Washington.

19. *Mechanics Magazine* (London), Jan. 1849.

20. See his British patent No. 6850 of 11 June 1835, and *Am. Mach.*, Aug. 4, 1888.

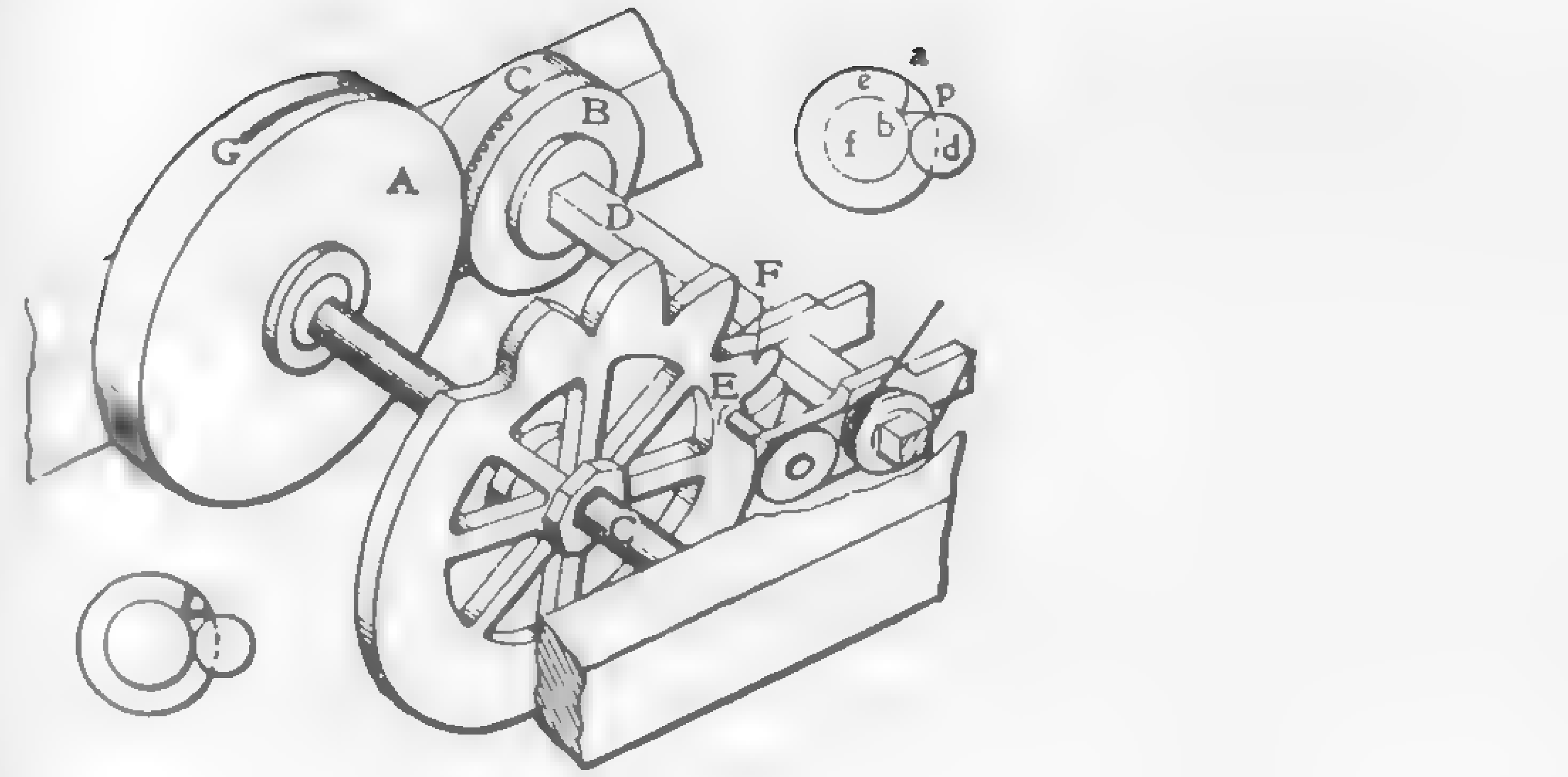


FIG. 20. BASHWORTH'S GEAR-GENERATING MACHINE, 1842
(*Mechanic's Magazine*)

bing cutter by a train of gears in exact ratio without slippage. This was the first machine to generate involute teeth. Another machine using the molding-generating principle invented by Hindley for cutting the Hindley worm is described by Smeaton,[21] but the date is unknown. In 1856 Schiele took out a patent for a machine designed to cut spur gears with a hob, also having gearing to rotate the blank and the hob at the correct relative speed.[22]

It seems then clear that production-type gear-cutting machines embodying all the basic methods of forming the teeth had been developed by 1850. This meant that accurately cut gears—spur, bevel, and worm—were available in quantity to English, and to a lesser degree to American,[23] machine designers. These gears were rapidly incorporated in textile machinery[24] and also in heavy power machinery.

21. John Smeaton, *Miscellaneous Papers*, London, 1814, p. 183.

22. See his British patent No. 2896 of 6 Dec. 1856, and below p. 105.

23. For the Gay & Silver Machine of perhaps 1841 see *Machinery*, May 1896, p. 263. See also the very solid machine with index disc and tangent screw, capable of cutting spur and bevel gears up to six feet, in *Am. Mach.*, Nov. 19, 1908, p. 746.

24. See the gear-cutting machine of prior to 1853, designed by W. B. Bement for the Lowell Machine Shop, manufacturers of textile machinery. Description and drawing in Oliver Byrne, *Handbook for the Artisan, Mechanic, and Engineer*, Phila., 1853.

It is not the purpose of this paper to consider the influence of all this development on conditions of production, the factory and the workers, only to provide a sound technical basis for such economic and sociological conclusions as others may draw.

Evolution of the Production Machine, 1850-1910

It would be unprofitable to list all the numerous types of gear-cutting machines and gear cutters developed from 1850-1910, to say nothing of the hundreds of mechanisms invented to put the principles of cutting gear teeth into effect. The intention is then to consider only those machines which represent a basically new application of the mathematical analysis to a production-type machine. To try to do more would be to bog down in a quagmire of antiquarian trivia.

FORMED-CUTTER TYPES

The formed-cutter gear-cutting machine had two principal types, both of which appeared in the early period as we have seen. One type was essentially a shaper and used a formed single-point tool. For very large gears the greater ease of supporting and manipulating a heavy gear blank upon a horizontal bed early produced slotter variations of this type.

The other type of formed-cutter machine, which was to prove in this period the most important for commercial production of spur gears, was the multiple cutter, or formed milling-cutter gear-cutting machine. As the period developed, this kind of gear-cutting machine was more and more widely used for all spur and spiral gears, except for large gears which were made on templet machines.

The rotary, multiple, formed-tooth milling type of gear-cutting machine was, as I have tried to show, a natural evolution from the wheel-cutting engine of the clock maker.

All that was necessary in this period was to adapt it to the manufacture of industrial sizes and quantities of gears.[25] Figure 18a shows Whitworth's machine of 1834-44 for cutting spur and bevel gears. The machine embodies no principles that we have not seen before, but is itself substantial and clearly a production machine, even though the drive for the cutter was by a gut band and spur wheels—hardly suited to heavy or continuous work. The cutter was fed in by hand. The later Whitworth machine shown in Figure 18b is clearly a full industrial machine in size and strength. Its drive is by a flat belt through a worm-and-wheel mechanism, although it still has hand feed for the cutter.

By 1851 we have the machine of Messrs. Shepherd, Hill, and Spink shown in Figure 21. Although the cutter

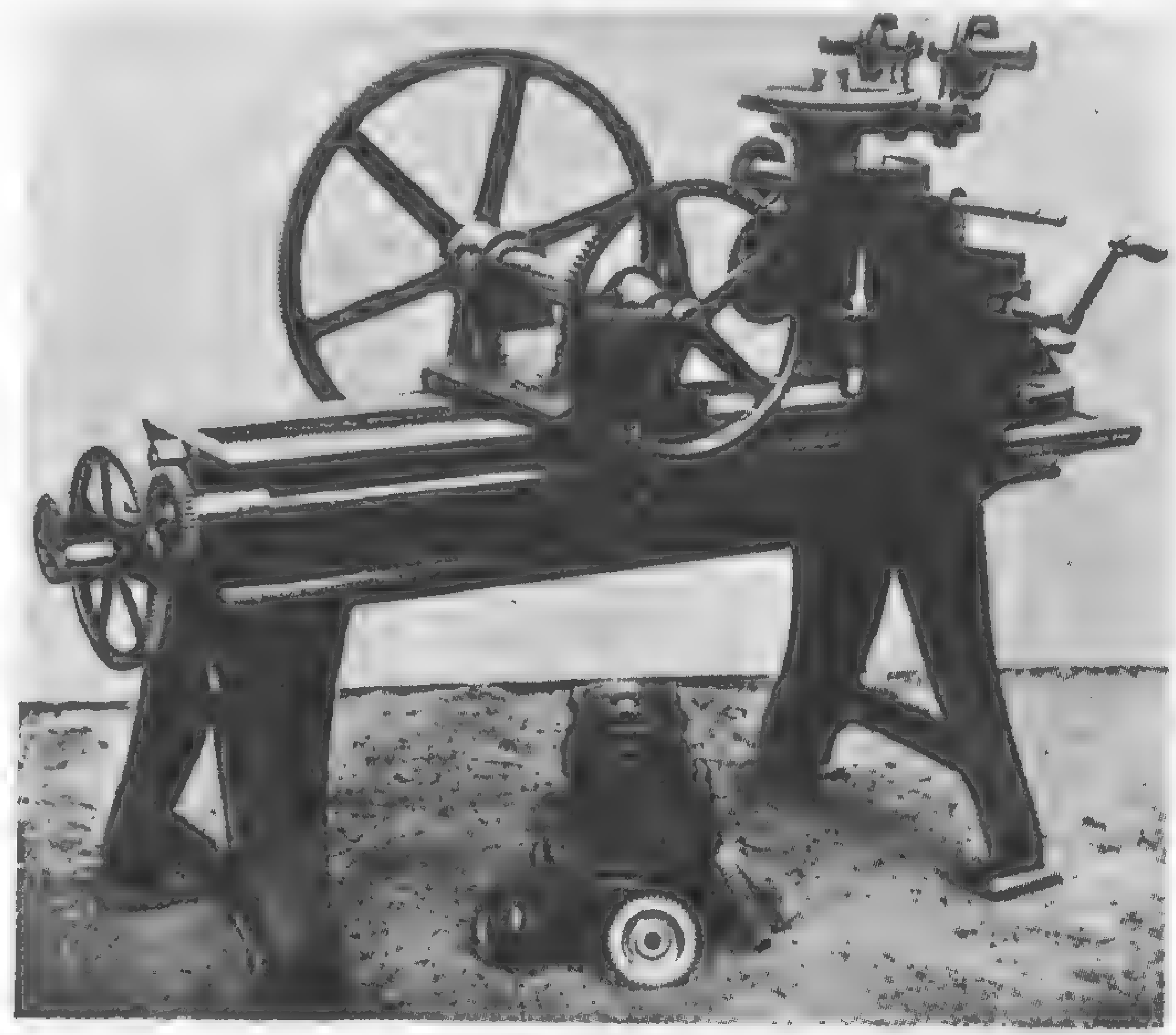


FIG. 21. SHEPHERD, HILL AND SPINK MACHINE, 1851
(*Inst. Mech. Eng.*)

25. James Brindley (1716-1772), of Bridgewater and Grand Trunk Canal fame, had constructed a machine for making gears for a silk mill about 1755. See Samuel Smiles, *Lives of the Engineers*, London, 1861, Vol. I, pp. 320, 326, 377.

drive is by an endless rope and acts through spur gearing, the machine is over-all a production machine. In addition, it had self-acting feed. Indexing was by a hand wheel, but the correct divisions were provided by change gears.

This development climaxes with the appearance in 1855 of Joseph R. Brown's "Precision" gear-cutting engine (Fig. 22).[26] This was not offered for sale by Brown and Sharpe; they used it only for manufacture of index plates to order and for manufacture of "precision" gears. Its divisions were copied from those of a Troughton and Simms dividing

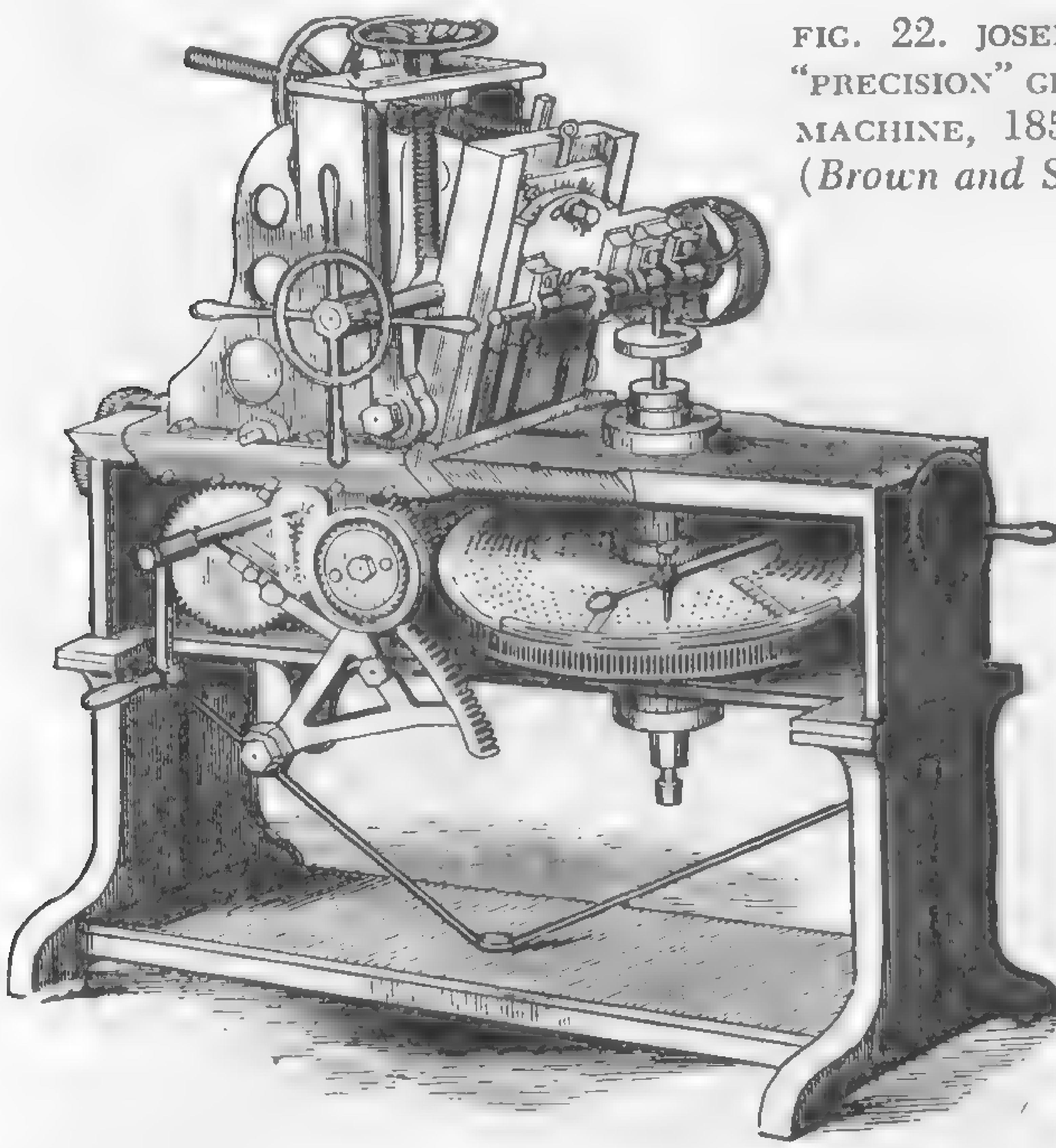


FIG. 22. JOSEPH R. BROWN'S "PRECISION" GEAR-CUTTING MACHINE, 1855 (*Brown and Sharpe*)

26. The illustration is from a dated wood cut in the possession of Brown and Sharpe. The machine is illustrated and described in their earliest catalogue of May 17, 1867. Brown and Sharpe still have one of these "front machines" in their collection, and one is exhibited in the Chicago Museum of Science and Industry. For a photo of one of these still in use in 1913, see *Amer. Mach.*, Oct. 16, 1913, p. 625. The author has been unable to find any information about Brown and Sharpe's machine referred to in their advertisement in the *Providence Journal*, April 22, 1833.

engine. It could, however, cut gears up to 37 inches in diameter and not coarser than 4 pitch. The description indicates that it was used for involute teeth.

Taken together with Brown's invention in 1854 of the backed-off formed milling cutter, with the fact that Brown and Sharpe put sets of these cutters on the market at least as early as 1867, we find in this machine all the principal features of the non-automatic milling-type gear-cutting machine.

TEMPLET TYPES

The formed-tool types of gear-cutting machine had a number of serious problems, especially for cutting the true epicycloidal teeth. For teeth of the epicycloid-hypocycloid form the necessary undercut of the inner flanks of the teeth made the rotary milling-type formed cutter impossible. This cutter could, however, be used with teeth having the involute form, or with epicycloidal teeth having radial inner flanks. But for the larger sizes of gears the formed milling cutters were far too expensive. Both single-and multiple-point formed cutters had an additional serious objection in common, the problem of resharpening. The accuracy of the teeth cut with formed cutters depended, of course, upon the accuracy of the form of the cutters themselves. This meant that resharpening had to be done most carefully and accurately to a templet or on some sort of pantograph machine, except for Brown's patented cutters.

Many machine shops of this day which did not have a milling machine would have a shaper or a slotter. There were therefore many advantages in adapting an indexing head and some sort of templet control of the tool to a shaper or a slotter. Then a true single-point tool could be used to cut gear teeth; such cutters were easily sharpened with little skill or trouble. The basic templet principle is shown in Figure 11b. The tool is guided by the rollers riding on the templets.

The machine designed in the 1860's by the clock maker Potts is shown in Figure 23. This is the first attempt to plane

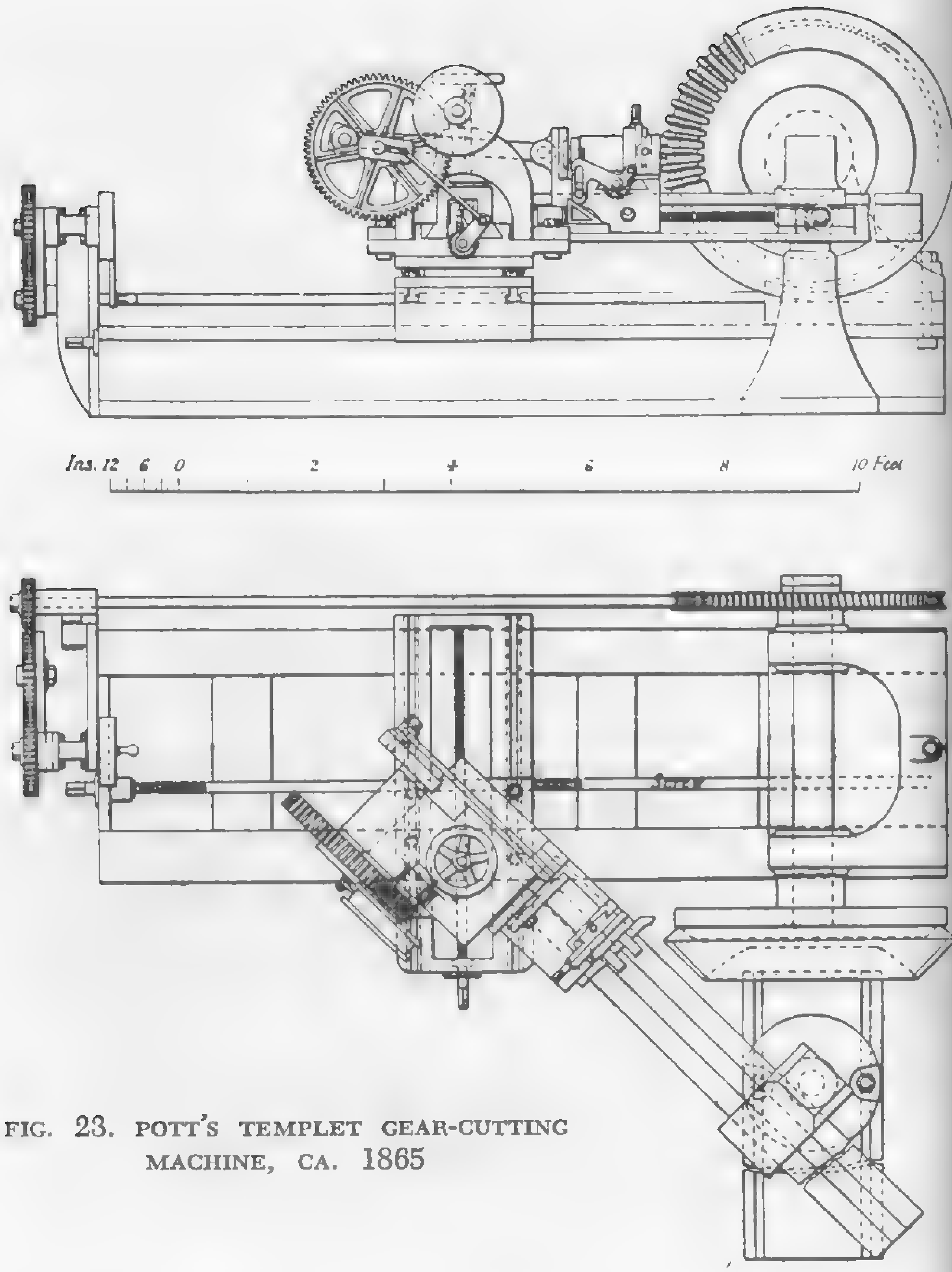


FIG. 23. POTT'S TEMPLET GEAR-CUTTING MACHINE, CA. 1865

both spur and bevel gears from a templet. The tool was carried on a reciprocating slide, and controlled in such a way as to form the desired tooth. In Potts' machine there were two templets alternately controlling a single cutter, one for each side of the tooth.

At the Centennial Exhibition of 1876 George H. Corliss showed a machine especially constructed to make the large bevel gears used in the huge Corliss engine which supplied power for the Machinery Hall. It used the templet principle, and attracted wide attention among engineers.[27]

The outstanding manufacturers of the templet-type machine have been the Gleason Company, of Rochester, New York. A large early one of these for cutting spur gears is shown in Figure 24. This machine could plane wheels up

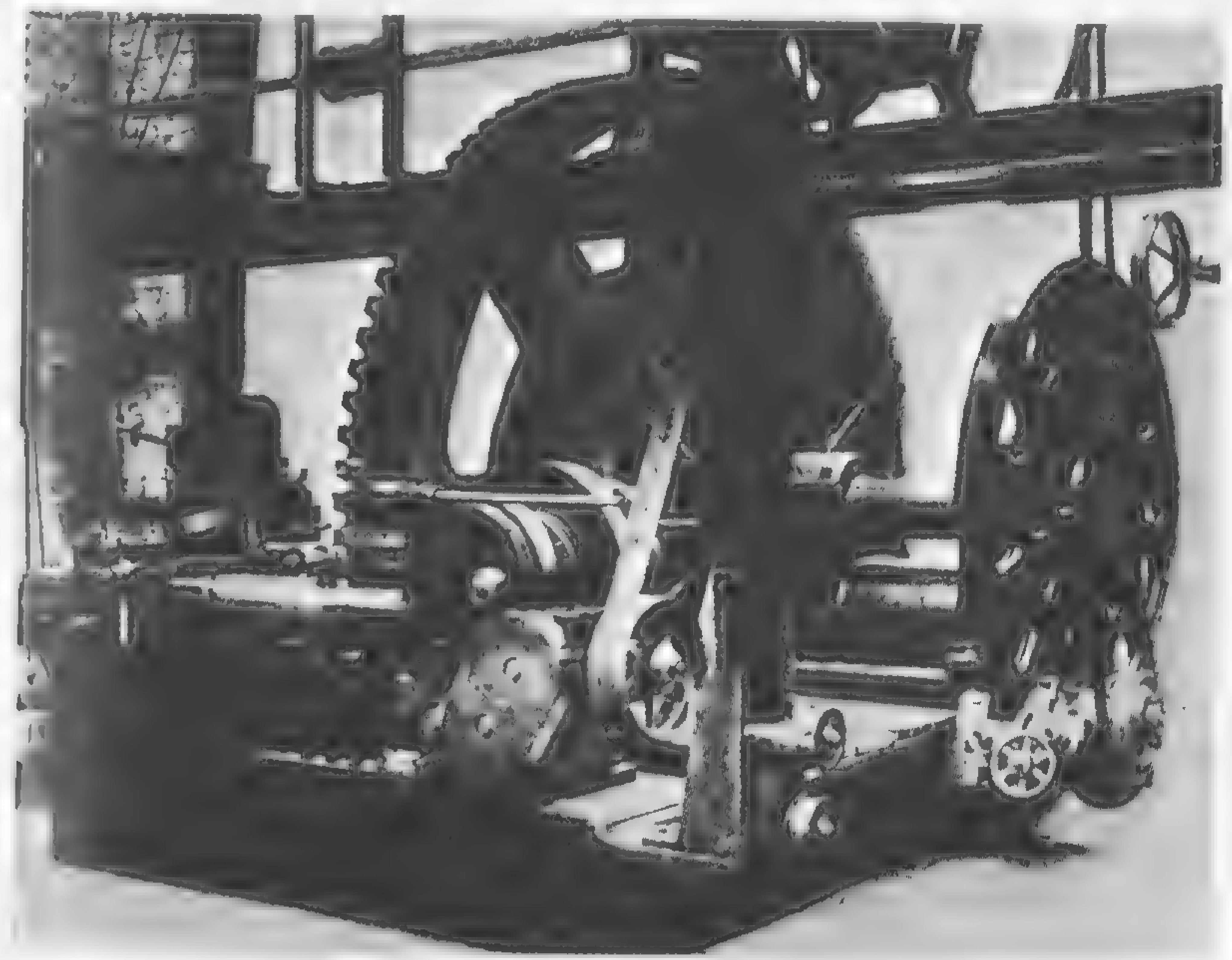


FIG. 24. EARLY GLEASON SPUR MACHINE, TEMPLET SYSTEM (*Inst. Mech. Eng.*)

to 15 feet in diameter. The Newton Company produced floor-plate gear-cutting machines of the templet type of even greater capacity (Fig. 25). The large gears cut on templet machines are usually first cast to dimensions which will give a minimum amount of metal to be removed.

27. *Machinery,* June 1898, p. 299. See his patent No. 6161 of Mar. 10, 1849 (*sic!*).

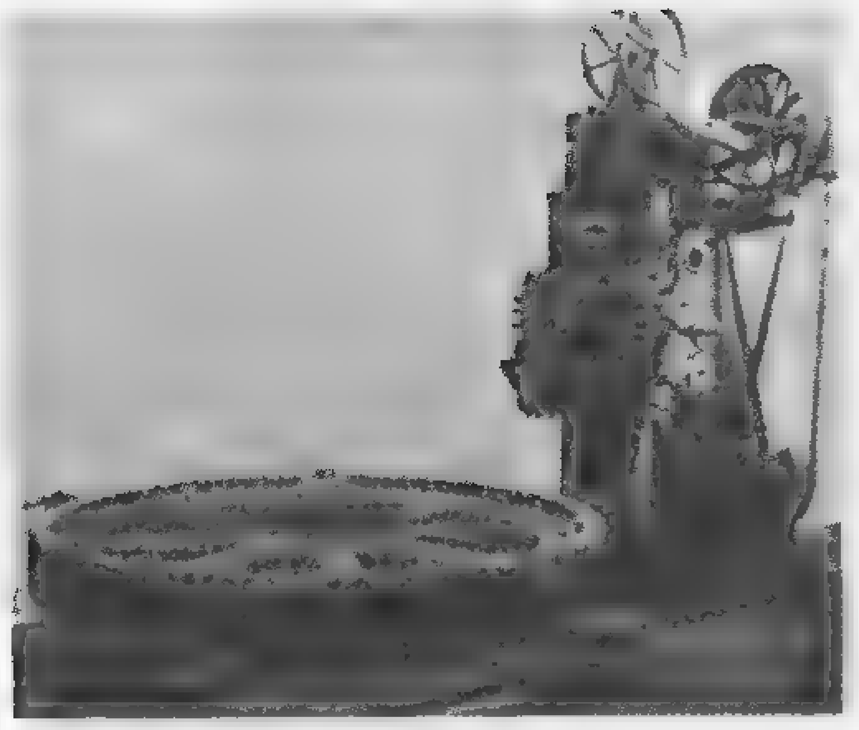


FIG. 25. FLOOR-PLATE GEAR-CUTTING MACHINE, TEMPLET SYSTEM (*Halsey*)

The Gleason Company also put out a templet-controlled machine for planing bevel-gear teeth (Fig. 26), the templets themselves being produced on a generating machine.[28] A templet-type machine using the grinding method for cutting the teeth of hardened steel gears was brought out by the Leland and Faulconer Company.[29] The templet principle has even occasionally been combined with generating principles.[30] With the widespread use of hobbing machines, the templet type is rapidly going out of use.[31]

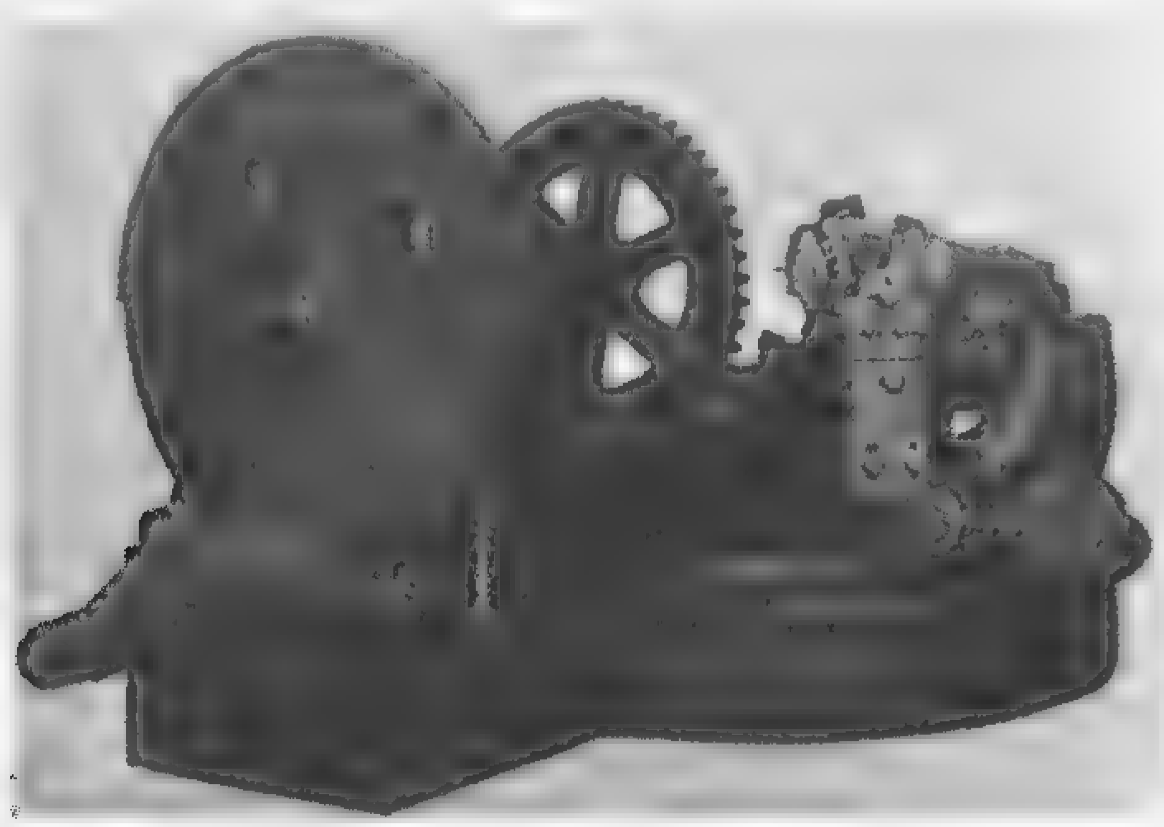


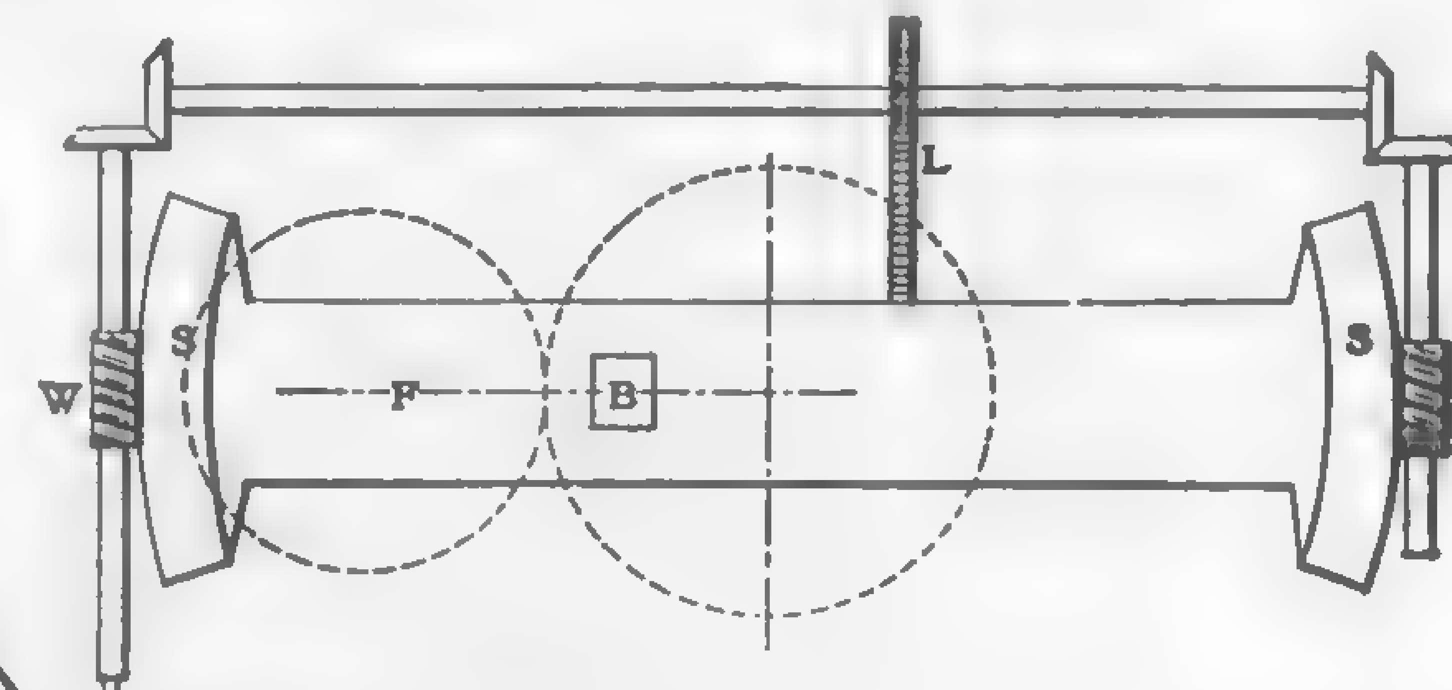
FIG. 26. GLEASON BEVEL-GEAR CUTTING MACHINE, TEMPLET SYSTEM, 1899 (*Halsey*)

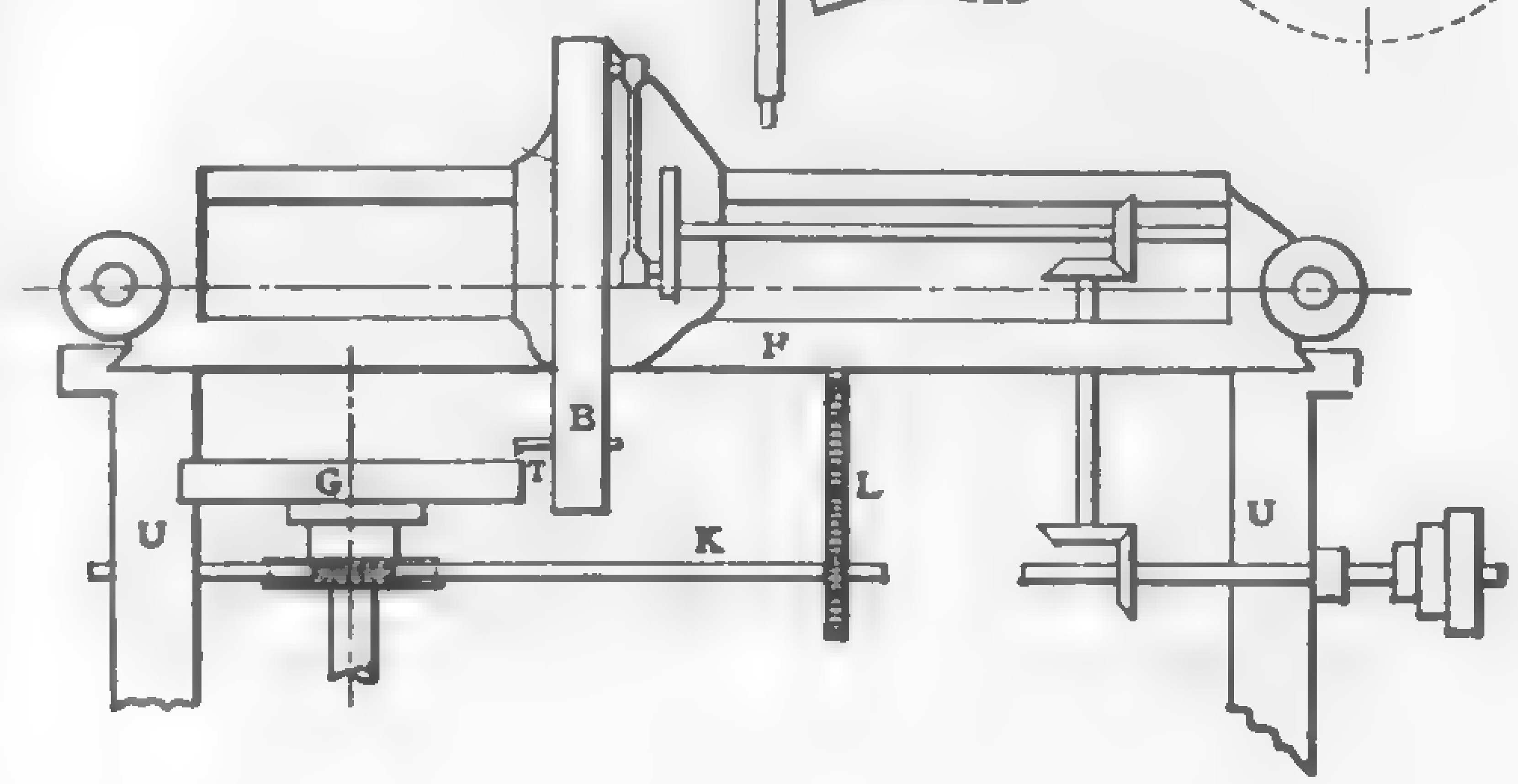
28. *Am. Mach.*, Nov. 8, 1900, p. 1058.
29. *Am. Mach.*, June 29, 1899, p. 589.
30. See John Hunt's British Patent No. 13,146 of 20 June 1850, and C. D. Rice's machine in *Am. Mach.*, May 10, 1900, p. 440.
31. But for Continental practice see Fred J. Miller, "Bevel Gear Cutting Machines at Paris" [1900], in *Trans. A.S.M.E.*, 1901, pp. 672-720; and *Am. Mach.*, Sept. 13, 1900, p. 881.

Although we have found generating types of gear-cutting machines already in use in the early period of production machines, they nearly all employed the molding-generating principle. But from 1840-1850 until 1884 the newer machines all worked by the describing-generating principle. The difference in the two methods depends solely on the form of the tool used. In the describing-generating method the cutting edge is theoretically a point and in practice is as nearly so as can be obtained (Fig. 11d). In the molding-generating method the cutting edge is formed into the side of a tooth, or a portion thereof, of a mating rack or gear (Figs. 11e and 11f). The relative motions necessary are often identical and in every case are very similar, whether an involute or an epicycloidal tooth is being generated.

One of the earliest of these describing-generating machines is that patented in 1859 by Lawson and Cotton.[32] This machine (Fig. 27), while ingenious, was imperfect. Although it could cut both spur and bevel gears, it did not form the hypocycloidal flanks required for interchangeable epicycloidal teeth.

FIG. 27. LAWSON AND COTTON'S GEAR GENERATOR, 1859 (*American Machinist*)





32. British Patent No. 2714 of 30 Nov. 1859.

It was necessary to develop a detailed application of the theory of gear teeth to this particular problem, before correctly designed machines of the describing-generating type could be produced. The first step in this direction was taken by E. Hagen-Thorn in 1872,[33] who showed how to generate involute teeth for both spur and bevel gears. He did not design an actual machine; he showed the basic principles by which a competent engineer could adapt a shaper for this purpose, by providing a lateral feed on the end of the ram. The tool is driven by the ram at right angles to the plane of the blank (Fig. 28a) and is simultaneously fed in the direction of the arrow. At the same time the blank is rotated in the same direction such that the cutting point is moved the same as that of a tangent line rocked around the pitch circle. Hagen-Thorn's tool would have two points, properly spaced so that both sides of the teeth are cut in one operation. The principle for bevel gears is shown in Fig. 28b, in which the bevel-gear blank is given a motion as if its conical pitch surface were rolling on a plane, the apex point remaining stationary on the plane where the vertical axis around which the conical surface rolls meets the plane.

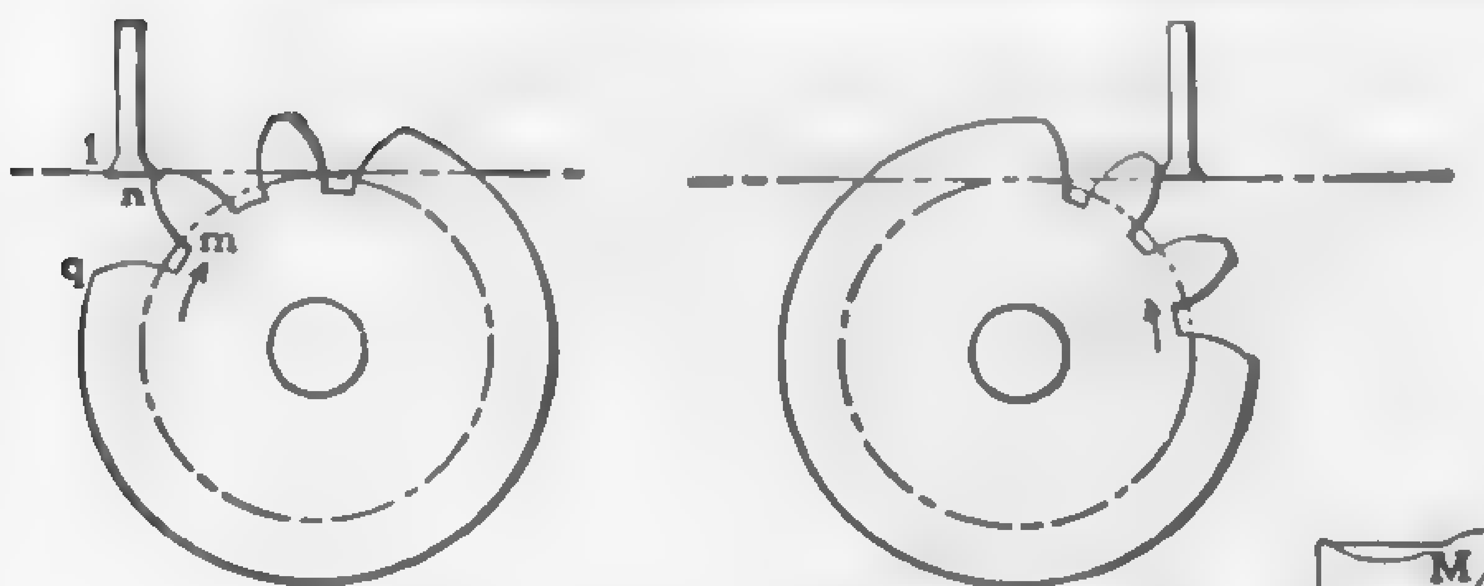
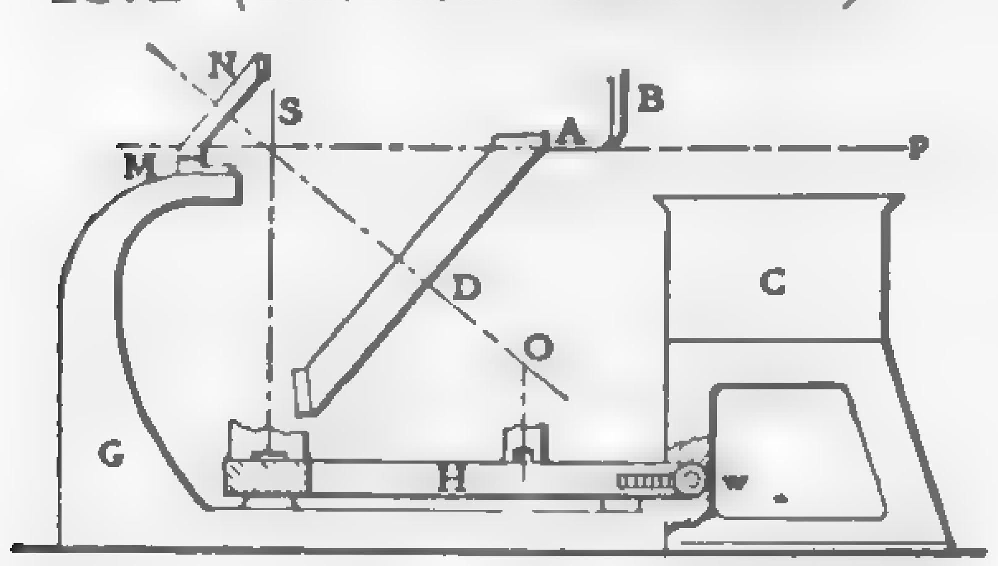


FIG. 28A. HAGEN-THORN'S INVOLUTE-TOOTH GENERATOR, 1872 *(American Machinist)*



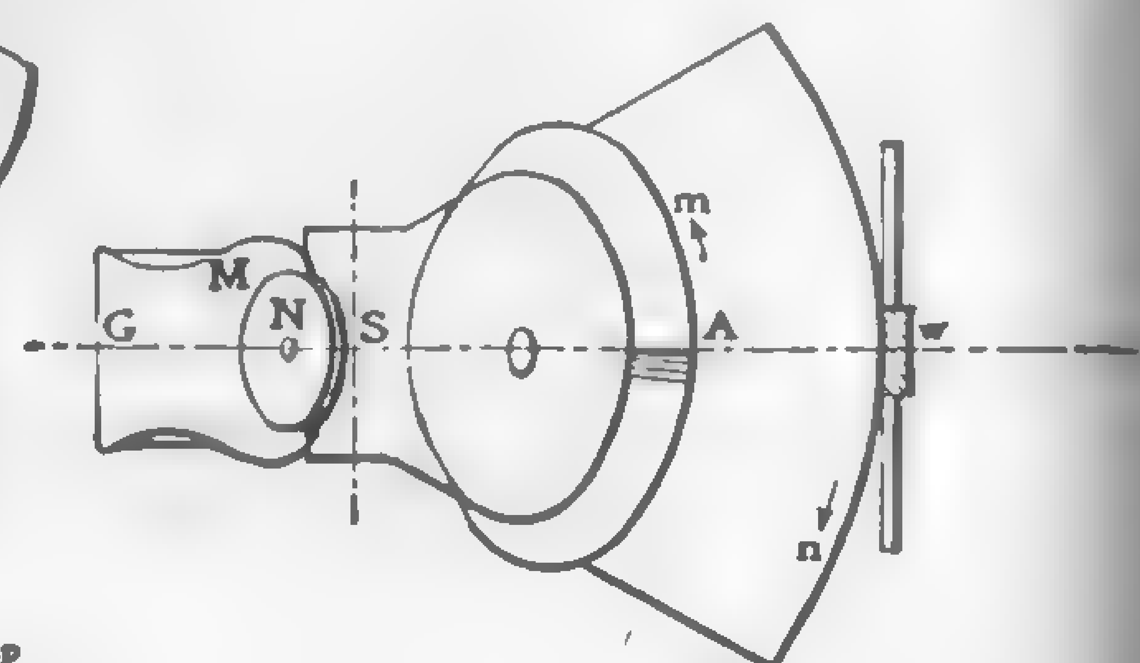


FIG. 28B. HAGEN-THORN'S INVOLUTE-TOOTH GENERATOR, 1872 *(American Machinist)*

33. E. Hagen-Thorn, "Über Herstellung möglichst genauer Zahnflanken," in *Z.V.D.I.*, 1872, pp. 353-368.

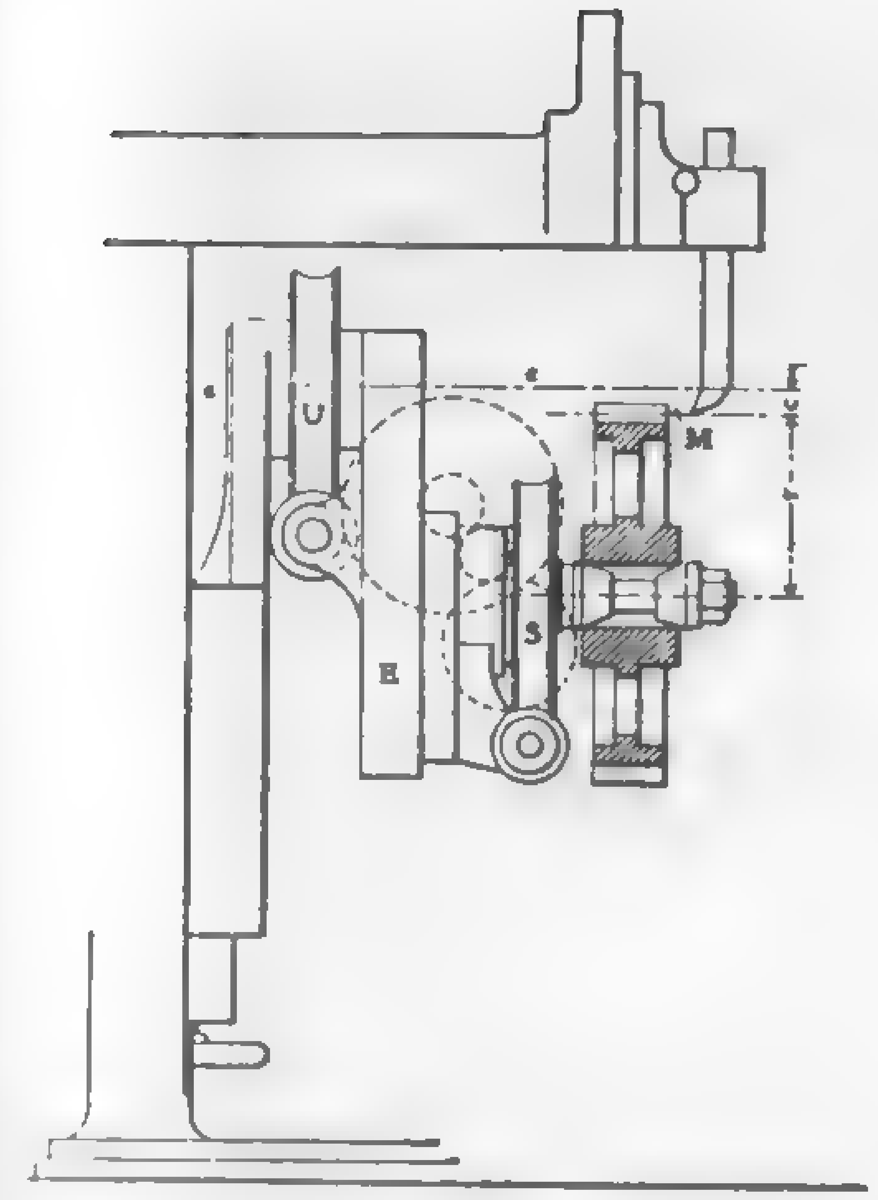


FIG. 29A. HERRMANN'S SHAPER FOR GENERATING EPICYCLOIDAL TEETH OF SPUR GEARS, 1877 *(American Machinist)*

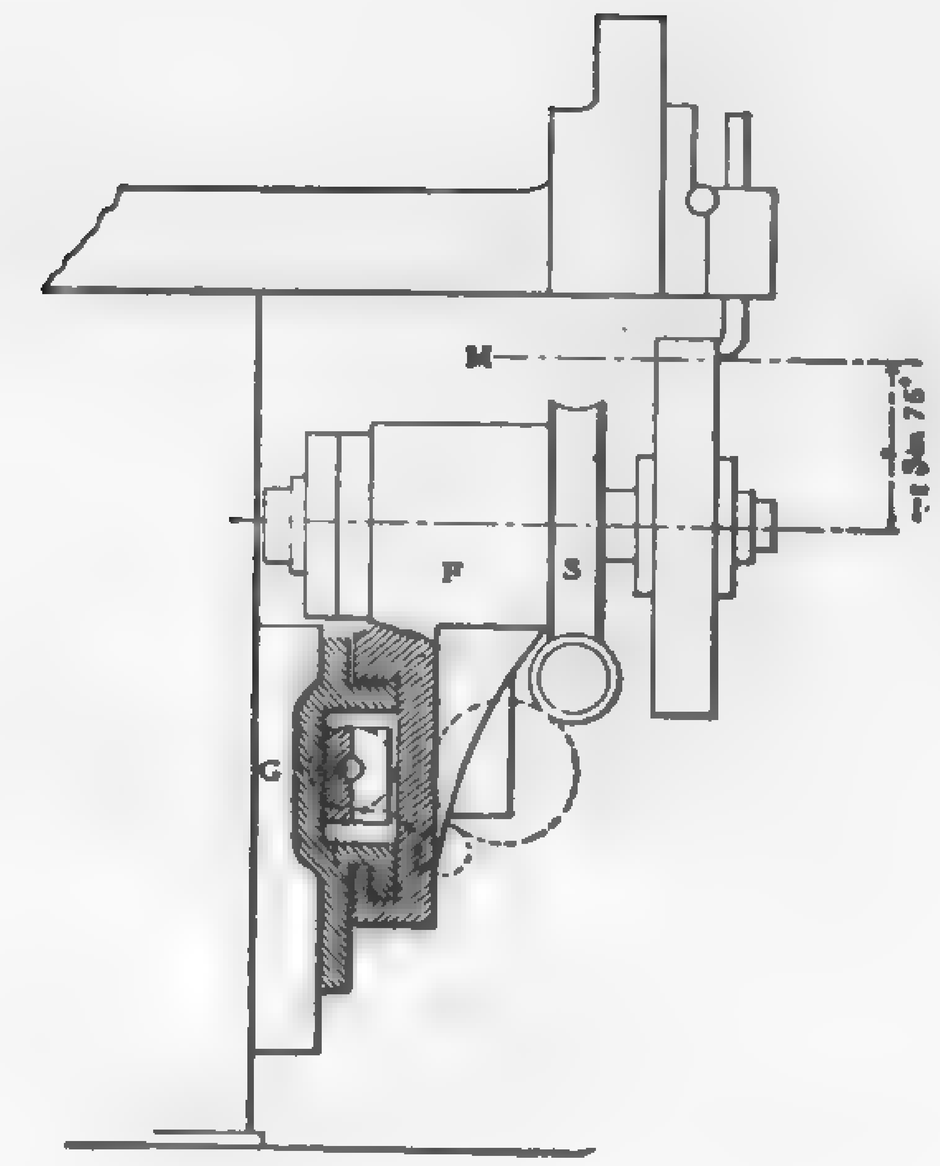


FIG. 29B. HERRMANN'S SHAPER FOR INVOLUTE TEETH, 1877 *(American Machinist)*

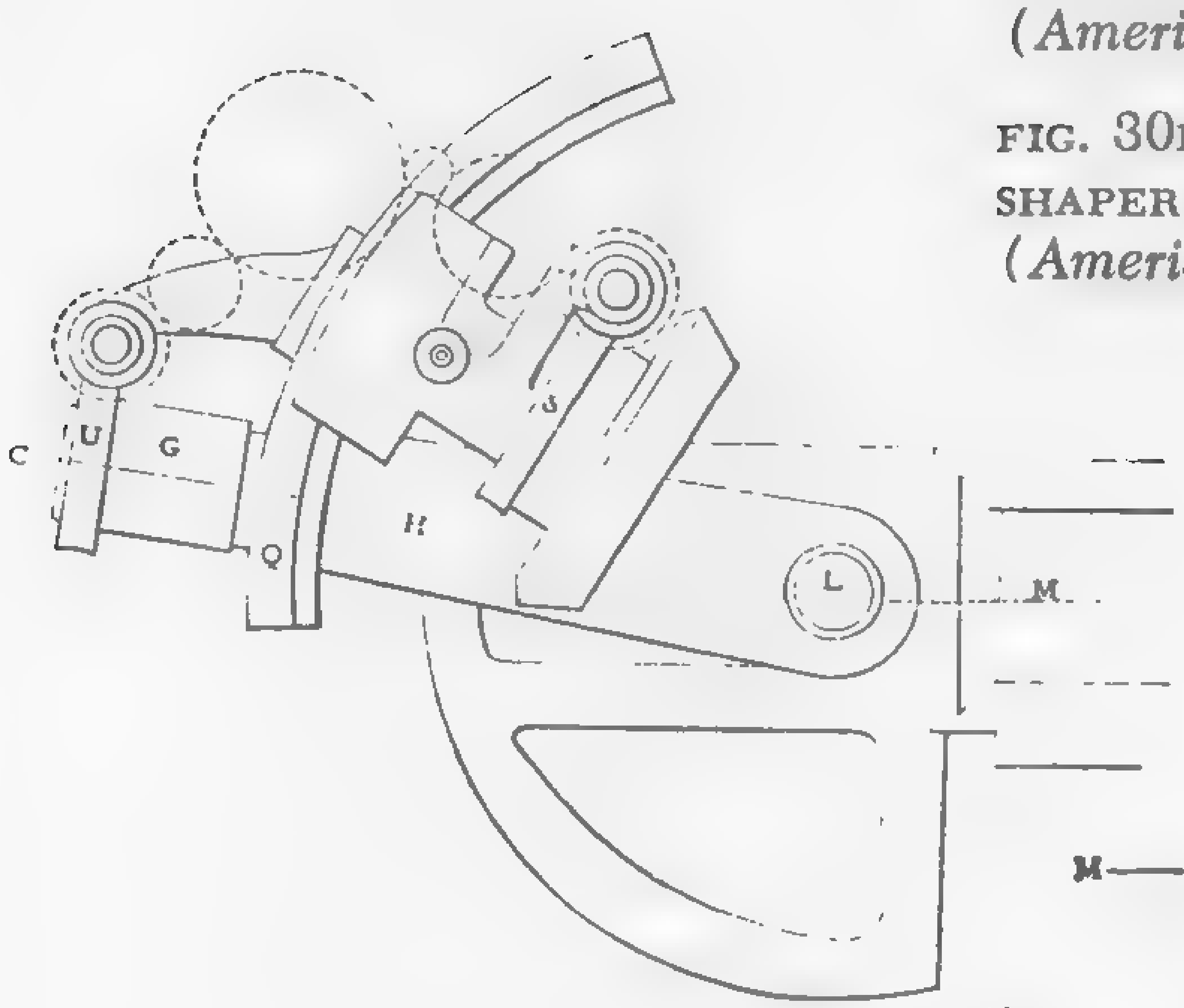
Another, more detailed and practical approach to the problem, also based upon careful theoretical analysis, was given by Herrmann in 1877.[34] Herrmann's method was more directly applicable to existing shapers to cut epicycloidal (Fig. 29a) or involute (Fig. 29b) spur gears. In all of Herrmann's machines a single-point tool is reciprocated as on an ordinary shaper. All the generating motions are given to the gear blank itself by the rather complex devices shown in the illustrations. For large gears Herrmann proposed to use a similar arrangement on a vertical shaper. He also designed means of using the shaper to cut either epicycloidal or involute teeth on bevel gears (Figs. 30a and 30b), and even showed how to cut skew gears by a similar method.

Two other describing-generating machines deserve mention as illustrating other means of accomplishing the same end. One is described in an Austrian patent of 1879 issued to C. Dengg & Company of Vienna.[35] This machine (Fig. 31) generates only epicycloidal teeth and only bevel

34. Gustav Herrmann, "Die Zahnflächen und ihre automatische Erzeugung," in *V.B.G.P.*, Berlin, 1877, p. 61.
35. Austrian Patent No. 657 of 4 July 1879.

FIG. 30A. HERRMANN'S EPICYCLOIDAL SHAPER FOR BEVEL GEARS, 1877 *(American Machinist)*

FIG. 30B. HERRMANN'S INVOLUTE SHAPER FOR BEVEL GEARS, 1877 *(American Machinist)*



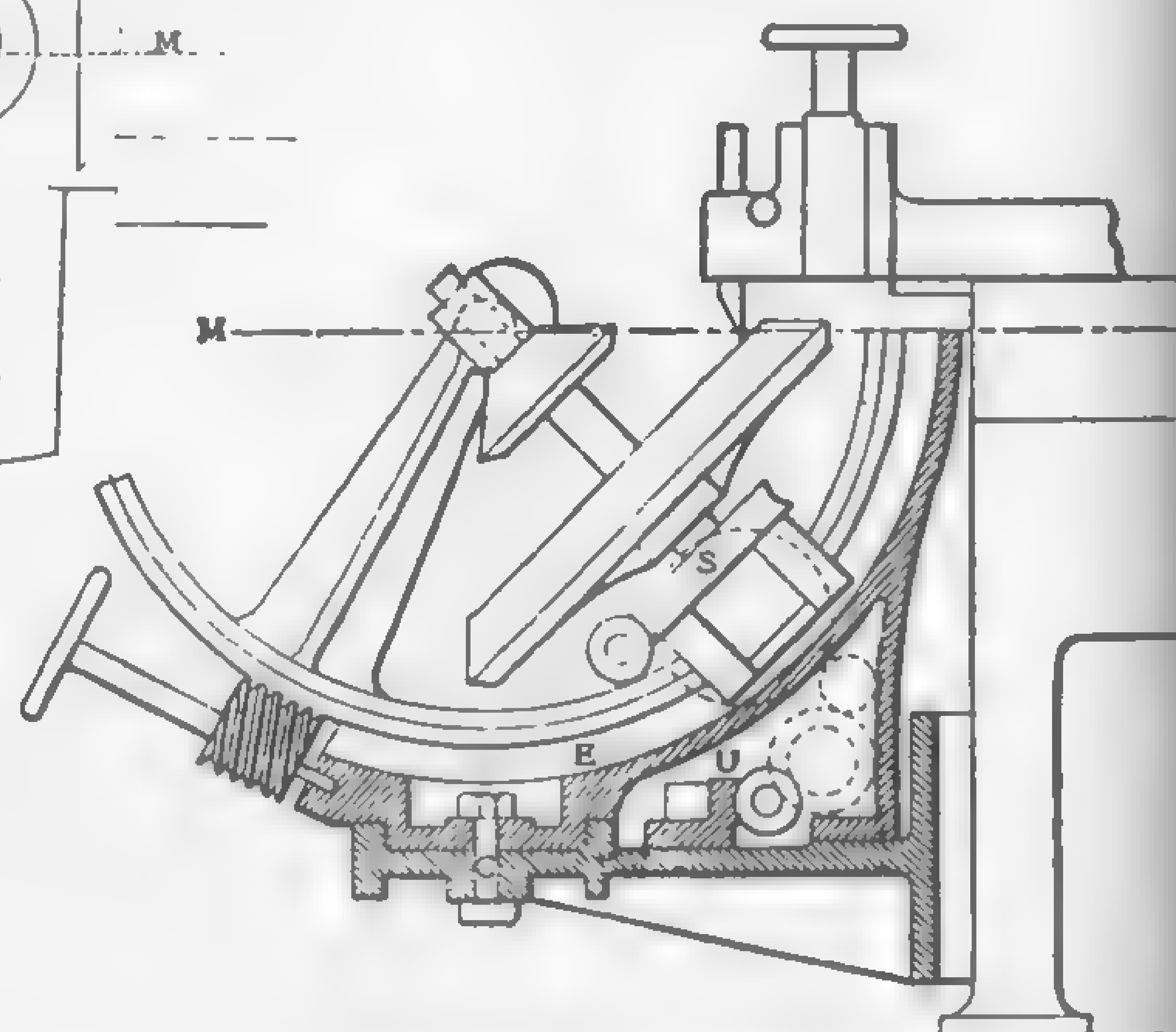
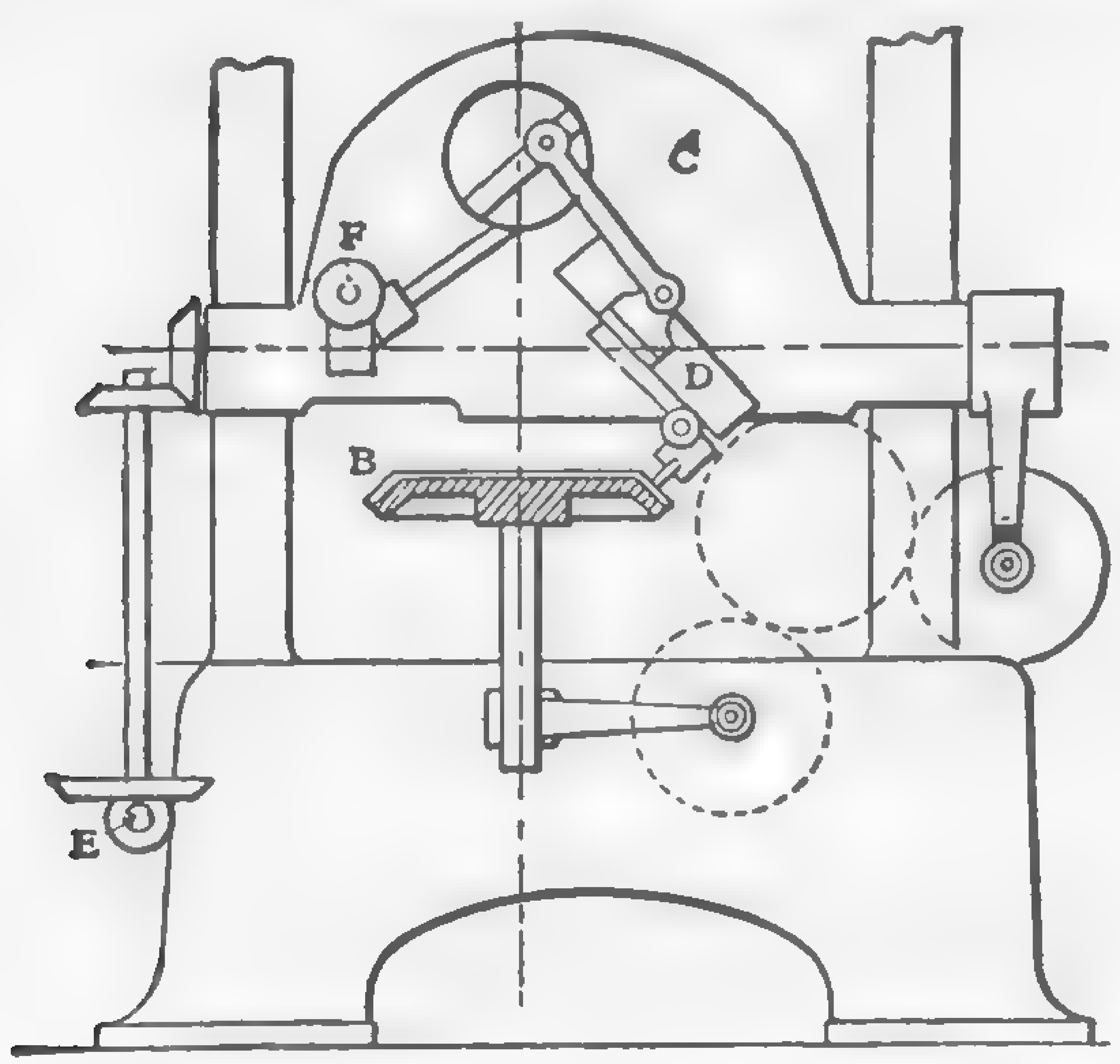


FIG. 31. DENGG EPICYCLOIDAL BEVEL-GEAR GENERATOR, 1879 *(American Machinist)*



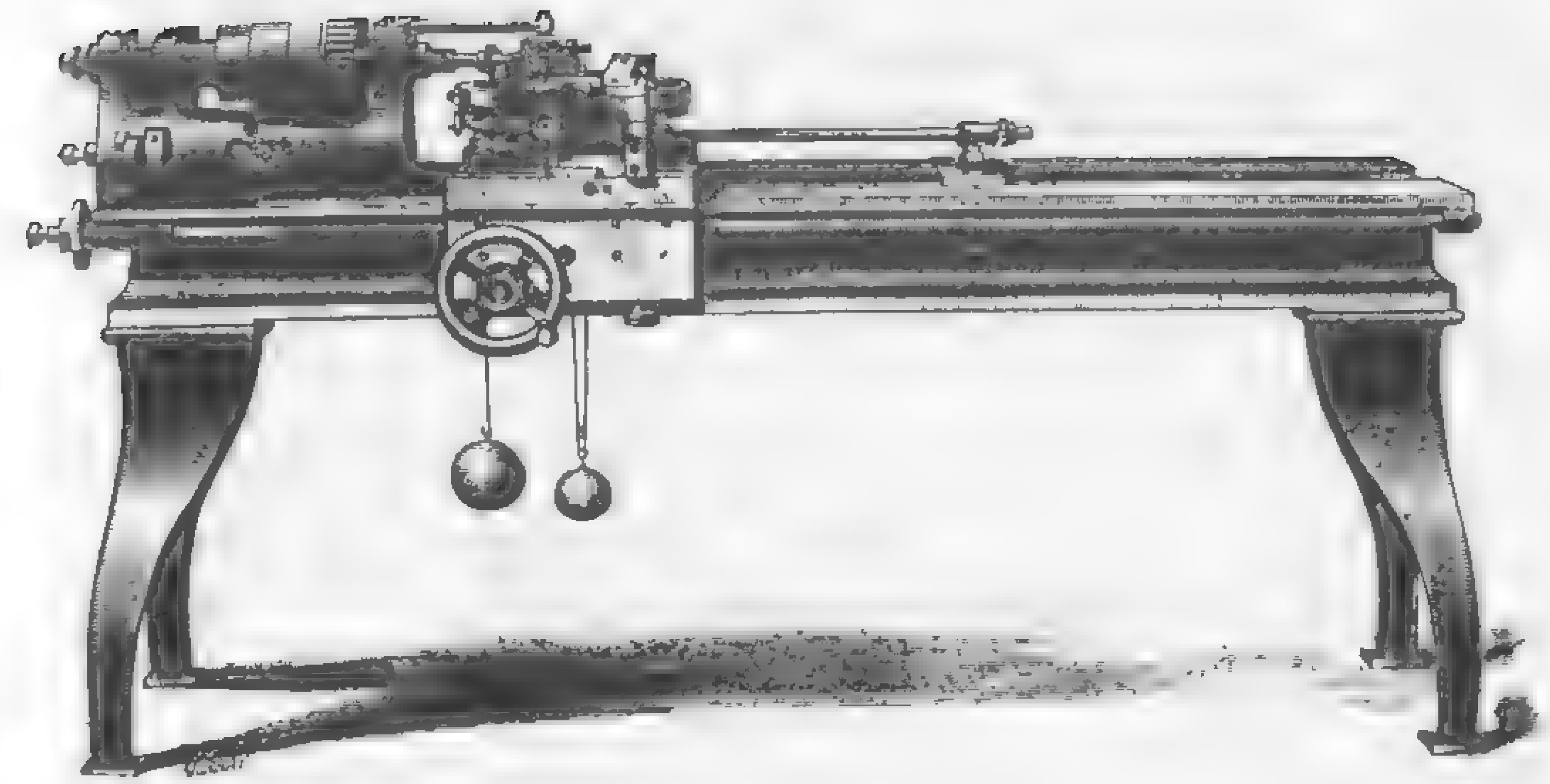
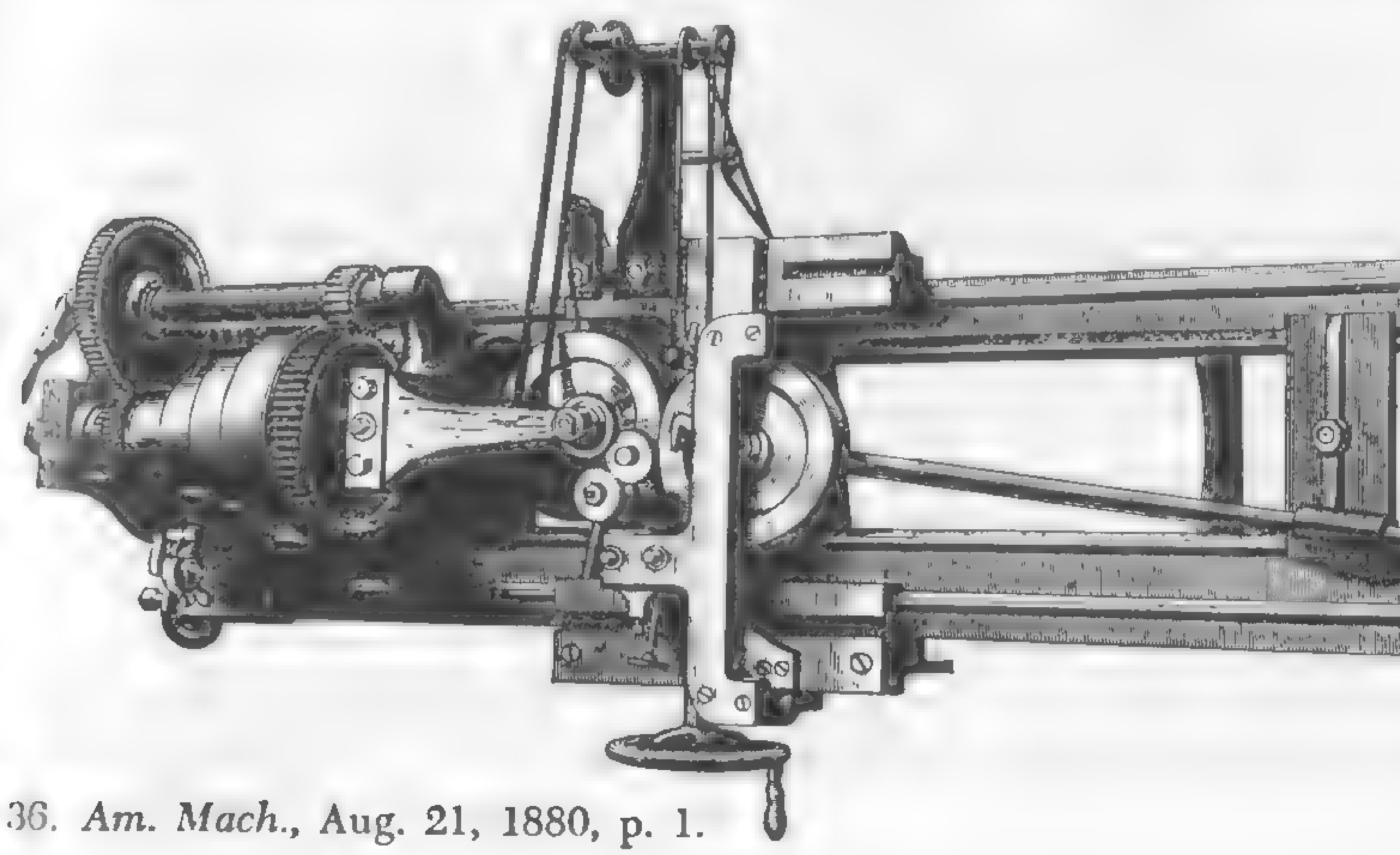


FIG. 32 PRATT & WHITNEY EPICYCLOIDAL ENGINE, 1880 *(Designed by Ambrose Swasey) (American Machinist)*

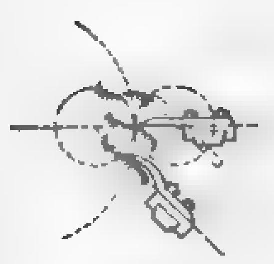
gears. In it the tool is guided along an element of the tracing curve, but the machine could not form hypocycloidal flanks. It is shown here as an example of the highly specialized types of gear-cutting machines sometimes developed.

One of the two significant American machines of this type is that of Ambrose Swasey called an epicycloidal engine.[36] This was built by Pratt & Whitney (Fig. 32). It used a small cylindrical cutter carried around first by a disc rolled on the outside of a ring representing the pitch circle and then automatically transferred to another disc moved as if rolling on the inside of the same circle. In a sense this was not really a production gear cutter, but more of an odonton



36. *Am. Mach.*, Aug. 21, 1880, p. 1.

like Beale's, since it was used only to shape the templets used in forming rotary gear cutters. But it is the only example of the describing-generating type using a cylindrical cutter.



The other American machine using the describing-generating principle was patented by George Grant in 1889.[37] It is shown in Fig. 33. A single-point tool reciprocates along a guide, *38*, which represents an element of the cone, *3*. The arm carrying the guide is rotated about an axis, the blank is rotated, both with such velocities as to give the tool a motion relative to the blank the equivalent of that of an element of the rolling-cone, *3*, relative to cone, *2*. This generates the epicycloidal faces of the teeth. The hypocycloidal flanks are shaped by the curved tool shown, now guided so that it becomes in effect an element of cone *4* rolling on the inside surface of cone. *2*.

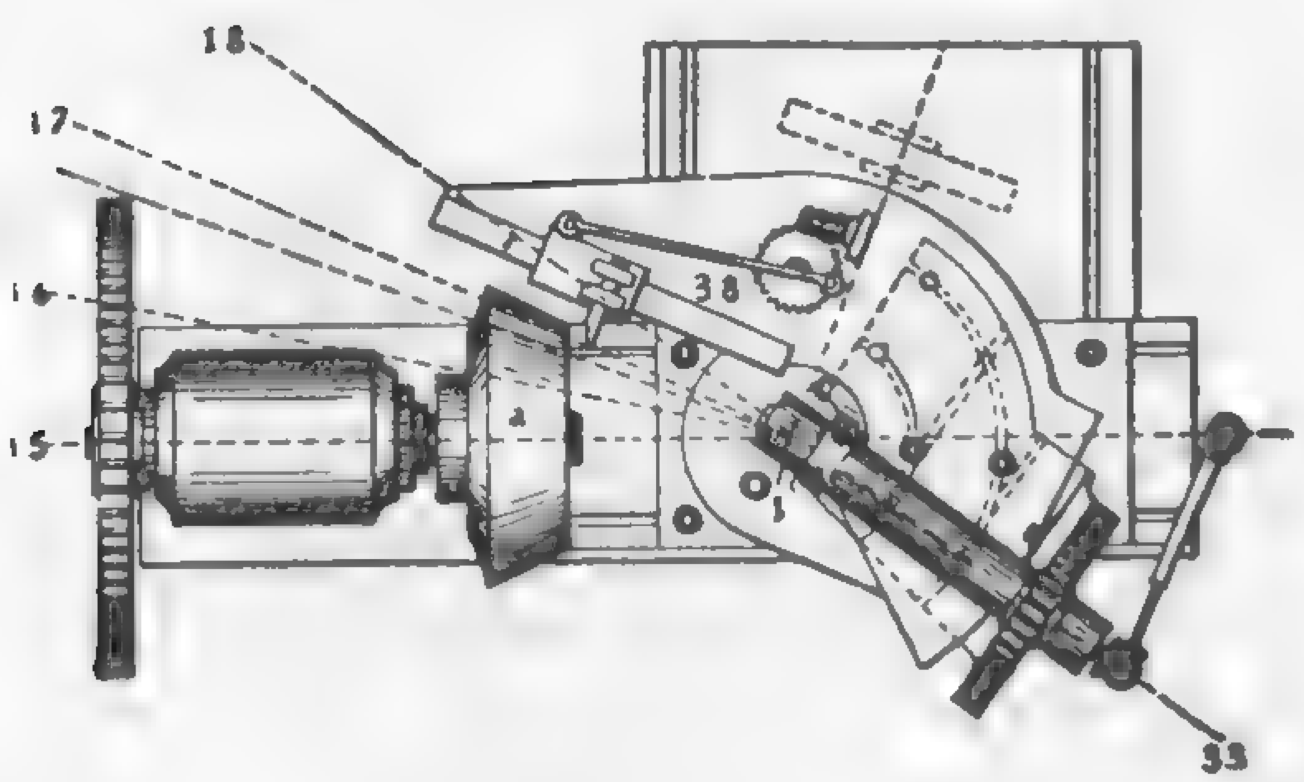
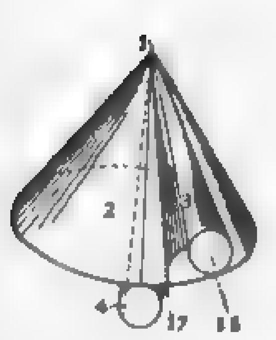


FIG. 33. GRANT'S BEVEL-GEAR GENERATOR USING THE DESCRIBING METHOD, 1889 (*American Machinist*)



Smith & Coventry of Manchester, England, exhibited at the Paris Exposition of 1900 a machine using the describing-generating method, but only shaping the teeth to approximate circular arcs.[38] This machine used two cutters working on opposite sides of the same tooth. The tools reciprocated along guides pivoted at the cone apex of the blank. The guides themselves were separated by a device using the swinging of the gear-blank carrier in a horizontal plane about a vertical axis through the cone apex.

37. See his Patent No. 407,437 of July 23, 1889.
38. See *Am. Mach.*, Oct. 4, 1900, p. 951, and Fred J. Miller, "Bevel Gear Cutting Machines at Paris" [1900], in *Trans. A.S.M.E.*, 1901, pp. 695-700.

It will be noted that machines of the describing-generating type originated mostly on the Continent and in the period from 1850-1884. This does not mean, of course, that machines of this type were not built elsewhere or later.[39]

MOLDING-GENERATING TYPES

The principle behind all the molding-generating machines grows out of Sang's theoretical analysis, which showed how a rack-tooth[40] shaped cutter could be used in an intermeshing method to produce interchangeable gears.

The first machine of this type was the invention of Hugo Bilgram[41] of Philadelphia in 1884 (Fig. 34). It was intended to produce only bevel gears, the demand for which had been greatly accelerated by the vogue for chainless bicycles. Bilgram had carefully studied the theory and mechanics of the molding-generating process, and his machine was an outgrowth of this investigation.[42]

The Bilgram machine used the shaper process for molding-generating, a method that came to be widespread, but it was the only one of these machines of any importance which used a single-point cutter. "*K* represents the blank to which both motions of translation and rotations are given; that is to say, it is, in effect, rolled upon its conical pitch surface under and past the tool *T*, which is reciprocated by the ram *B* across the face of the blank. As the spaces between the teeth of a bevel gear are tapering, one side of each tooth is finished first, the blank being indexed by the mechanism *LPN*. The tool can be adjusted laterally so as to bring the side at work into a radial plane of the pitch cone. The rolling motion is obtained by swinging the carrier *C*, on which the blank spindle *I* is adjustably supported in bearing *H*, around the vertical axis *YY*, in which lies also the apex of the pitch cone

39. See Ralph E. Flanders, *Gear Cutting Machinery*, New York, 1909.
40. A spur tooth could be used equally well, as in the Fellows gear shaper.
41. See his Patent No. 294,844 of Mar. 11, 1884; also *Am. Mach.*, May 9, 1885, pp. 1-2. A fine example of this machine is in the Ford Museum, Dearborn, Mich.
42. Hugo Bilgram, "A New Odontograph," *J. Franklin Institute*, Jan. 1882, pp. 1-14.

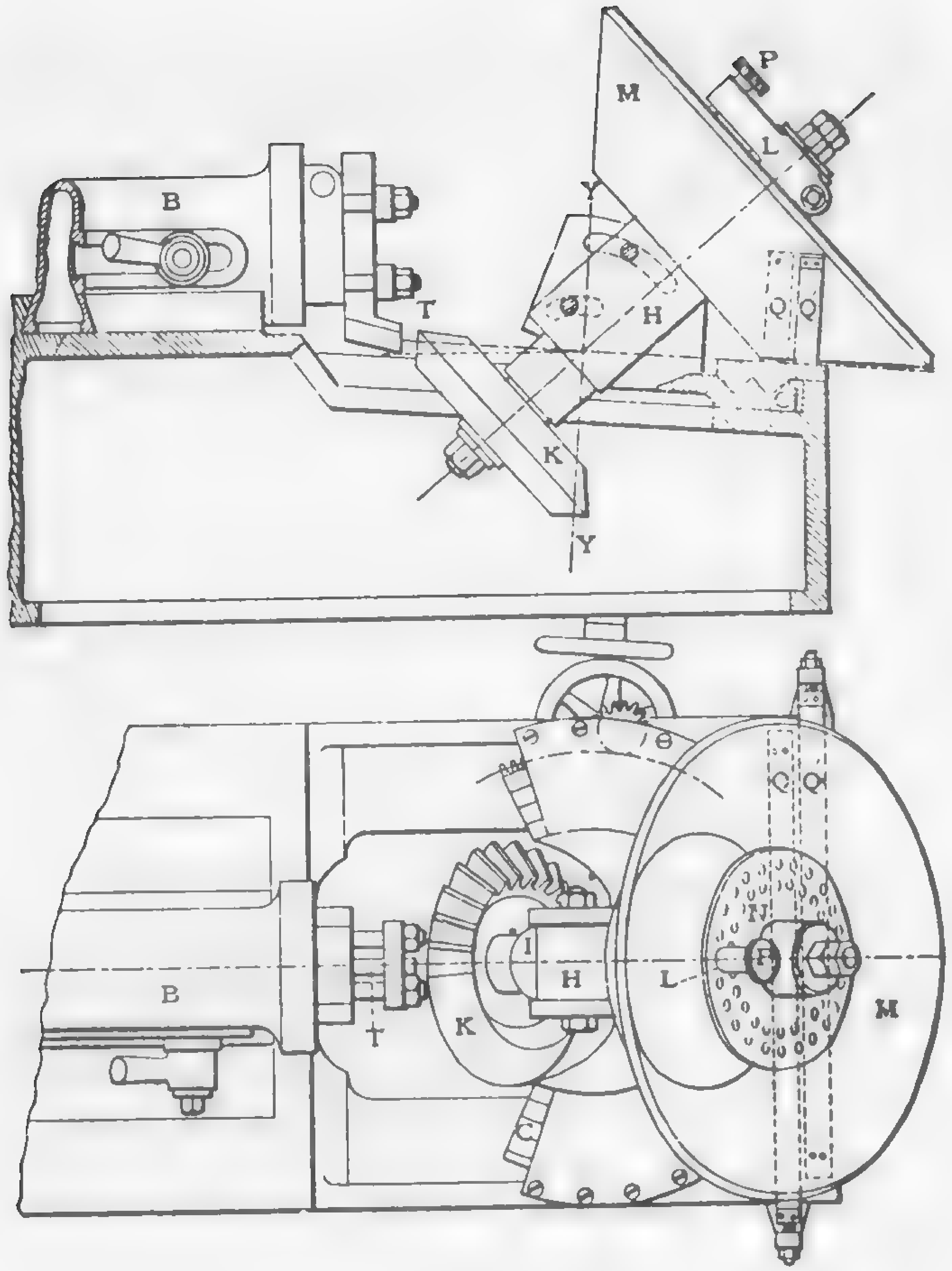


FIG. 34. BILGRAM'S BEVEL-GEAR GENERATOR, 1884
(*American Machinist*)

of the blank. This swinging movement causes a rotary movement of the blank on its axis by means of the two flexible steel bands Q and Q', which are attached at one end to the frame and pass in opposite directions around the cone M on the blank spindle representing part of the pitch cone of the blank. The cones are reversed because they lie on opposite sides of the axis YY."[43]

43. *Am. Mach.*, Aug. 13, 1903, p. 1152.

Bilgram's actual machine and the principle on which it generates the teeth are shown in Fig. 35. In 1900 Bilgram adapted the principles of this machine to one to generate spur and spiral gears.[44] This was an automatic gear-cutting machine operating by means of an indexing mechanism which rotated the blank after each stroke of the cutter, and a positive cutter lifter for clearing the tool on the back stroke, when the indexing takes place. The tool, therefore, made one cut in each space of the blank, then the blank is rolled slightly to make the next series of cuts. He later made the bevel-gear machine automatic.[45]

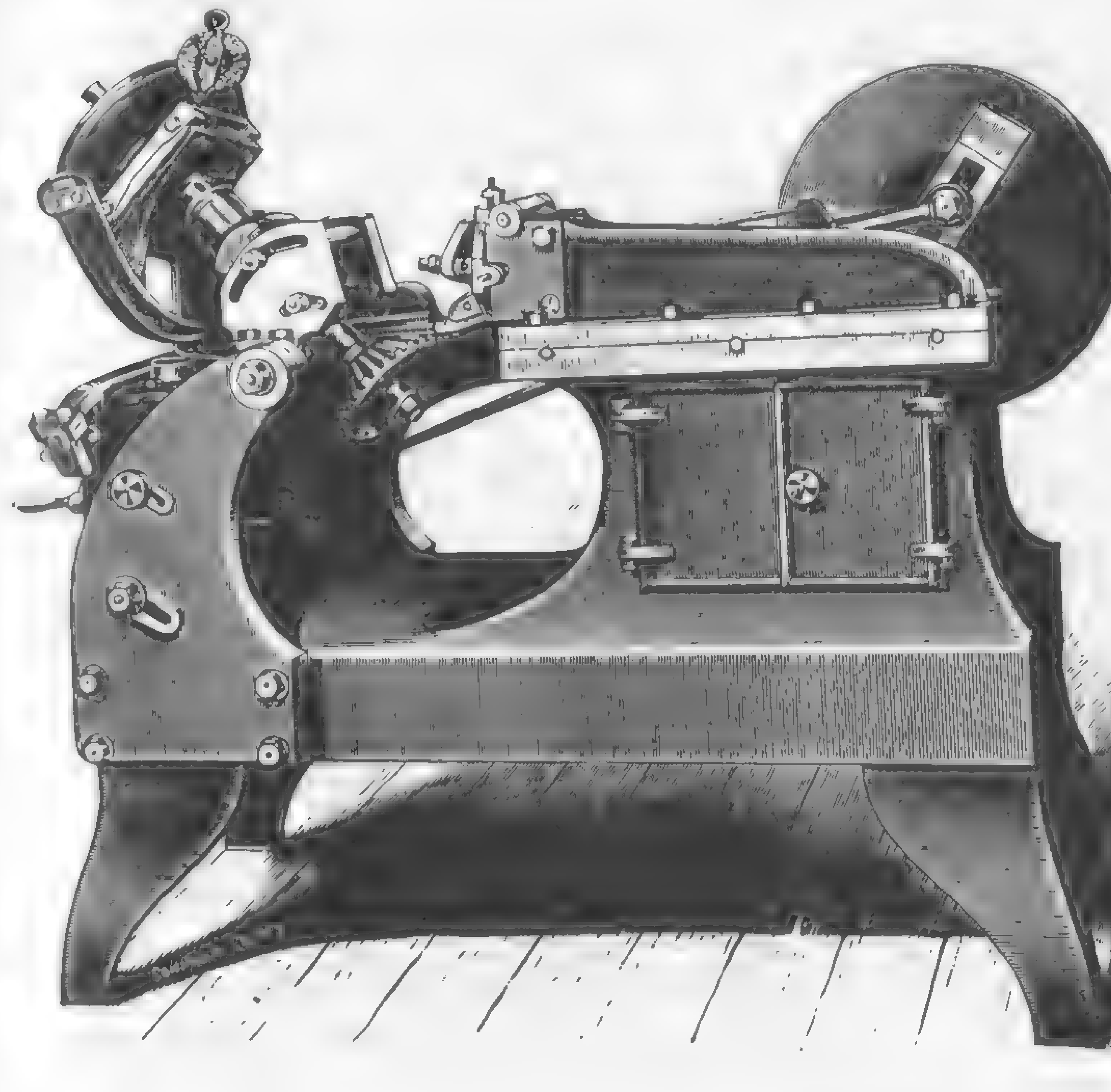


FIG. 35. BILGRAM'S BEVEL-GEAR CUTTING MACHINE, 1885
(*American Machinist*)

44. *Am. Mach.*, Jan. 31, 1901, p. 110.
45. *Am. Mach.*, Jan. 23, 1902, p. 114.
46. *J. Frank Inst.*, Aug. 1886, pp. 135-139.

Bilgram's invention was the subject of a careful examination by a committee of the Franklin Institute which finally awarded him a gold medal for his work.[46]

In his bevel-gear machine Bilgram had introduced his "octoid tooth." This entirely new tooth form is neither involute nor epicycloidal, but appears in all generated bevel gears. Sang had shown how the straight-sided involute rack tooth could be used to generate involute teeth. For the bevel gear the analogy to the rack tooth is the crown-gear tooth, but this does not have perfectly straight sides and is therefore difficult to make. To generate bevel-gear teeth Bilgram used a straight-sided crown-gear tooth. The resulting form of the bevel-gear tooth so generated was given the name "octoid" by George B. Grant since the locus of the point of contact between two gear-tooth forms is a curve resembling a figure of eight (Fig. 36).

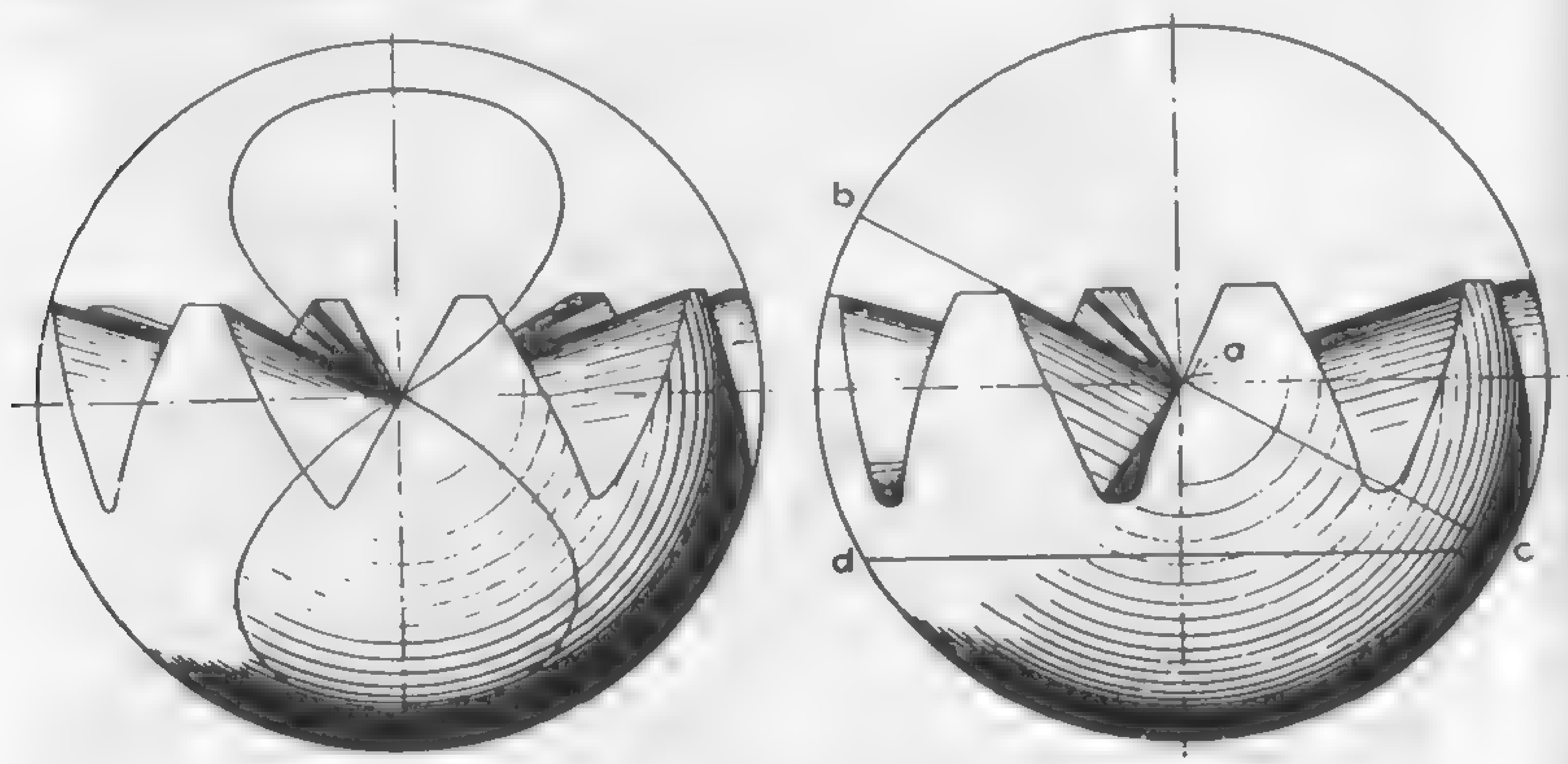


FIG. 36. BILGRAM'S OCTOID TEETH (*Halsey*)

Bilgram's machine used, of course, a single cutting edge on its tool. A molding-generating type machine using multiple-edge or milling-type cutters was invented by Ambrose Swasey and patented in 1885.[47] These cutters are shown at *K* in Fig. 37, and represent several teeth of a rack. The spur-

47. See his Patent No. 327,037 of Sept. 29, 1885, *Am. Mach.*, Nov. 13, 1890, p. 5, and Ambrose Swasey, "New Process for Generating and Cutting the Teeth of Spur Wheels," *Trans. A.S.M.E.*, 1891, pp. 265-274.

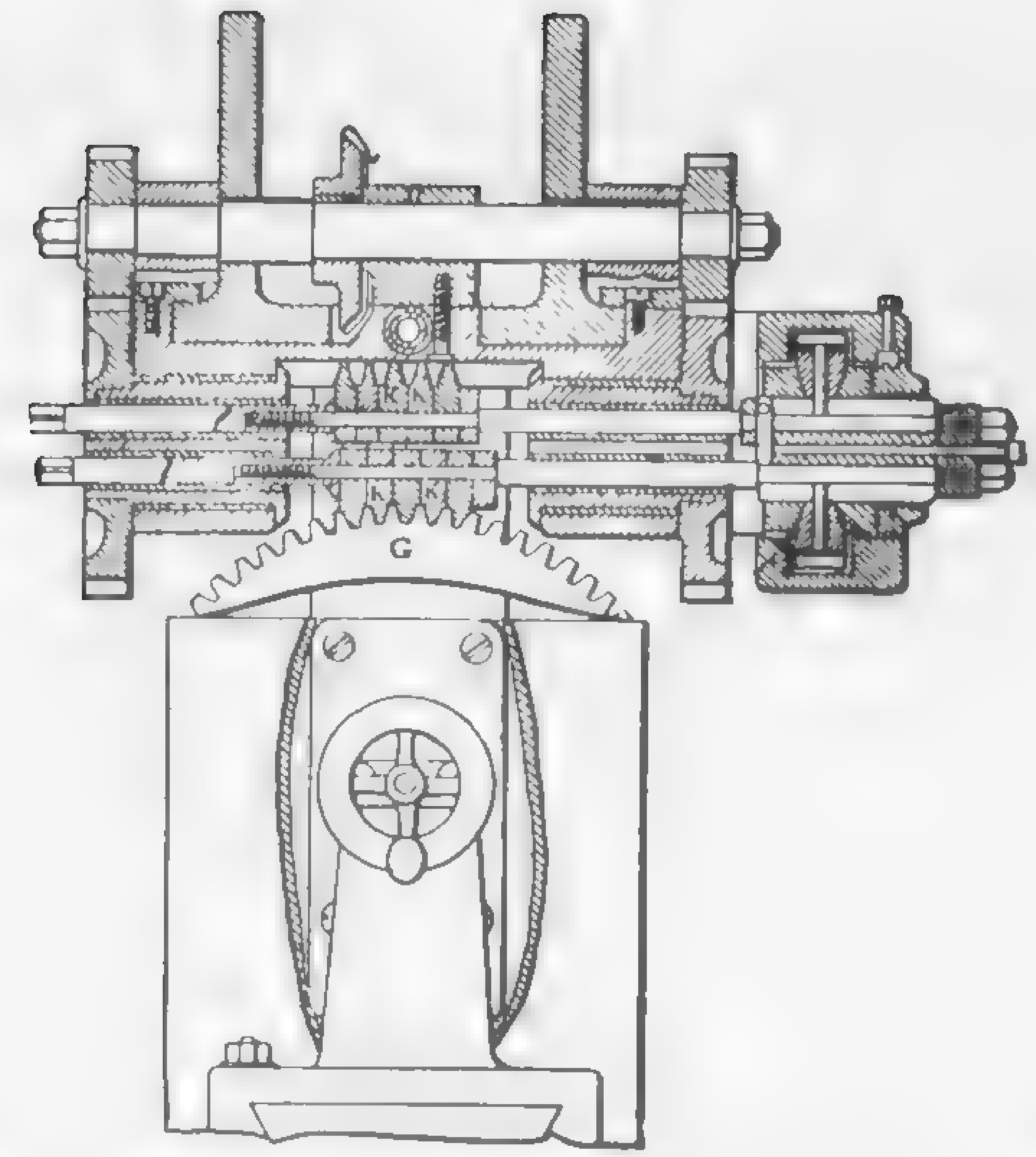


FIG. 37. SWASEY'S SPUR-GEAR GENERATOR, 1885 (*American Machinist*)

gear blank shown at *G* is carried on a horizontal arbor, and the rotating milling cutters pass across its circumference as in the ordinary formed-tooth gear cutter. However, in Swasey's machine the gear blank *G* is rotated so that the action is that of a gear and rack in mesh; this of course required a lateral feed of the cutters, which was accomplished by an ingenious and rugged cam mechanism. The cutters are divided in two parts along their axis, so that while one half is engaged and cuts the blank, the other half is moving backward to be ready to engage a new portion of the blank.

A similar machine was patented in 1896 by H. C. Warren,[48] but this used a gang of solid cutters greater in length than the circumference of the pitch circle. The cost of such a cutter prohibited its use in practice. Even Swasey's machine was too complex for general use.

Another bevel-gear machine utilizing the molding-generating principle is George B. Grant's (1849-1917) Planoid Bevel-Gear Generator (Fig. 38).[49] This produced the

48. See his Patent No. 559,011 of Apr. 28, 1896.
49. *Am. Mach.*, June 7, 1894, p. 2; see his Patent No. 512,189 of Jan. 2, 1894.

planoid teeth invented by Grant, which have plane flanks, usually radial, and faces conjugate to the plane flanks of the mating gear. They are to epicycloidal spur teeth what octoid bevel teeth are to involute.

In Grant's machine the tool "t^2" reciprocates along the guideway while the blank is rolled out of engagement with it. This, in effect, causes it to roll on the pitch surface of the gear whose tooth flank the tool represents.

A vertical slotter-type machine developed by C. C. Tyler in 1895 is shown in Fig. 39b and its principle in Fig. 39a. It uses a tool of involute rack shape reciprocating on the slotter bar and at the same time fed across the bed tangentially to the blank which is at the same time rotated at the same linear speed by two flexible steel bands. This gives the same motion as that of a rack and gear in mesh.[50]

The reader has doubtless already noted that most of the gear-cutting machines based upon the generating principle were designed primarily to generate the teeth of bevel gears. The explanation for this is to be found in the practical difficulties of cutting them in any other way. Correct bevel-gear teeth cannot be cut by any of the formed-tool methods. They can, of course, be done on templet machines, but with great trouble and at slow speeds.

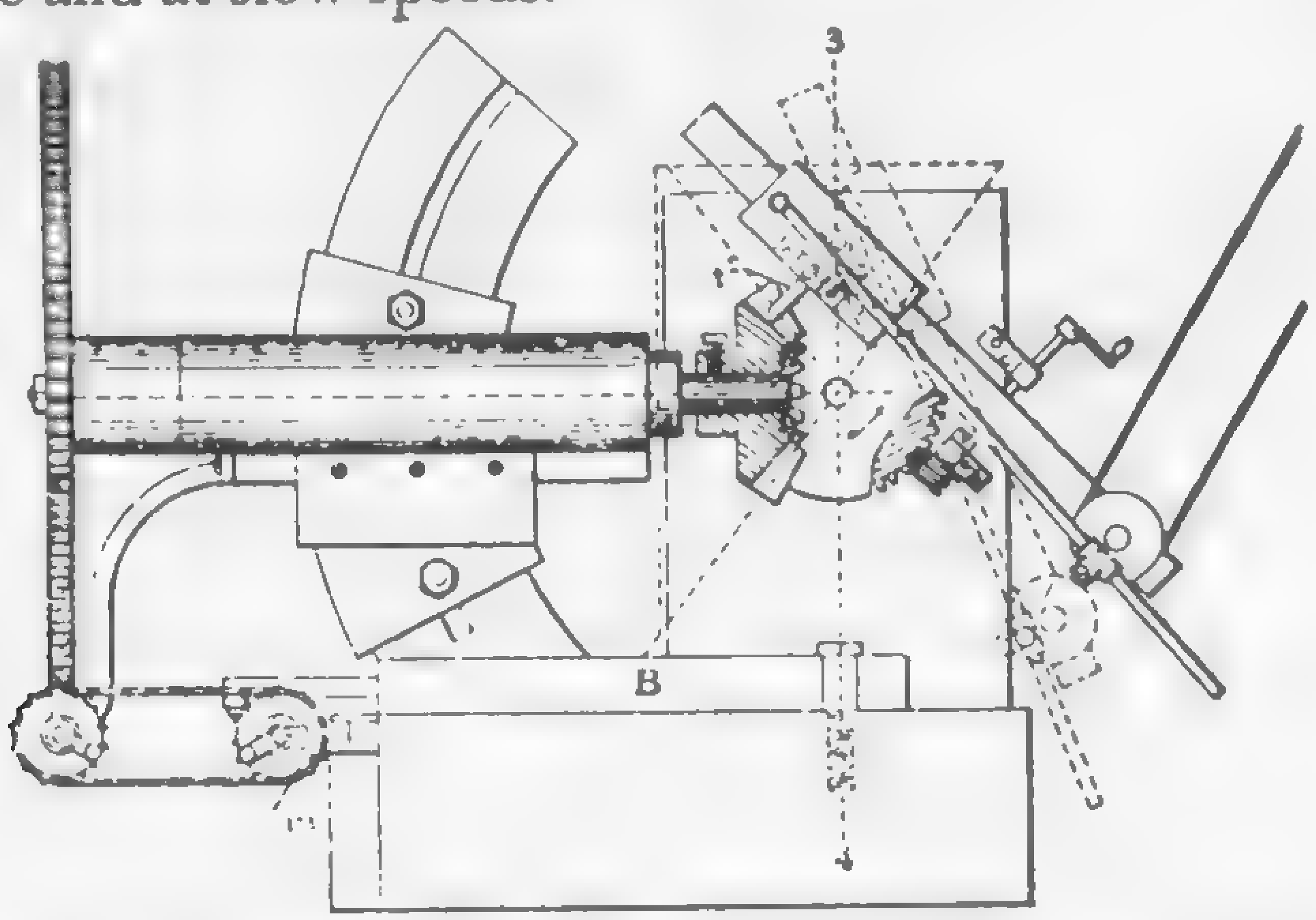


FIG. 38. GRANT'S PLANOID BEVEL-GEAR GENERATOR, 1894
(*American Machinist*)

50. See his Patent No. 551,065 of Dec. 10, 1895.

The principles and practical devices of the various generating-type machines inevitably led to a recognition of the advantages of the generating principle for cutting spur gears, especially for internal gears. The result was one of the most widespread and influential of the gear-cutting machines—one based upon the molding-generating principle—the Fellows Gear Shaper.

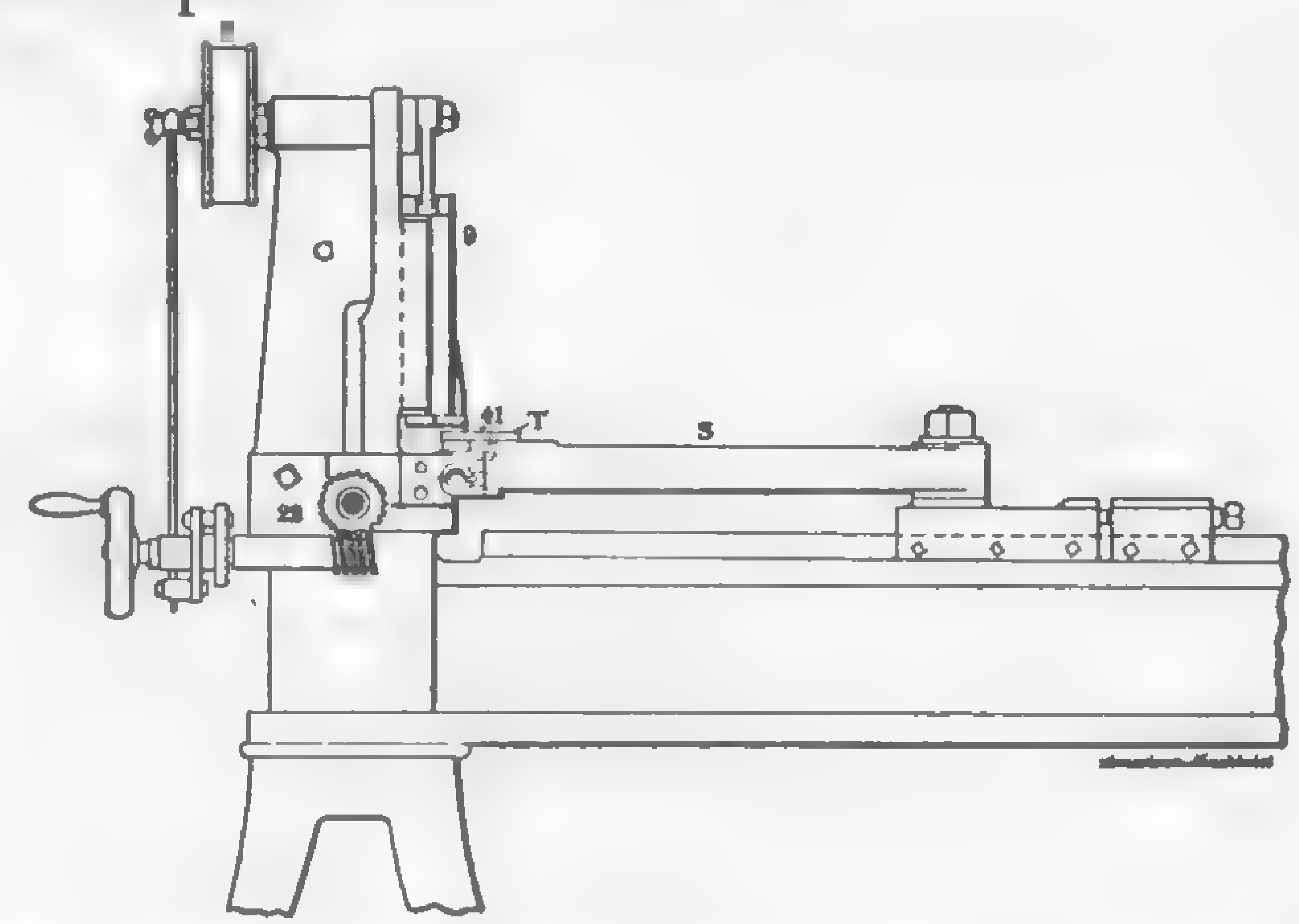


FIG. 39A. FORMATION OF A GEAR SPACE FROM A RACK TOOTH
(*American Machinist*)
FIG. 39B. TYLER'S INVOLUTE CURVE-SHAPING MACHINE, 1895
(*American Machinist*)

In Sang's day an accurate sharpened and hardened edge could be had only in the circle and the straight edge. By 1890 the grinding machine had progressed to the point where it was possible to generate teeth by this method in an already hardened gear cutter, as well as to finish gear teeth to a precision not hitherto possible.[51] This fact made the Fellows Gear Shaper, and its necessary counterpart the Fellows Cutter Generator, possible.

The Fellows Gear Shaper[52] was the invention of E. R. Fellows. So significant was it that in 1899 he was awarded

51. This story will be presented in a later monograph on the History of the Grinding Machine.
52. *Am. Mach.*, Dec. 7, 1897, pp. 915-919, and Feb. 15, 1900, pp. 153-155. See Fellows' Patents: No. 579,708 of Mar 30, 1897, for the shaper; No. 686,599 of Nov. 12, 1901, for the cutter grinder; No. 579,570 of Mar. 30, 1897, for the cutter.

the John Scott Medal by the Franklin Institute. This machine (Fig. 40a) was of the molding-generating type, but using a complete gear instead of the rack tooth as the generator. This gear is especially cut, hardened and then ground accurately to shape and its cutting edges given back and side clearance and top rake in the gear-cutter grinding machine. The grinding machine itself was a grinding disc which represents one side of a tooth of an involute rack, or a crown gear.

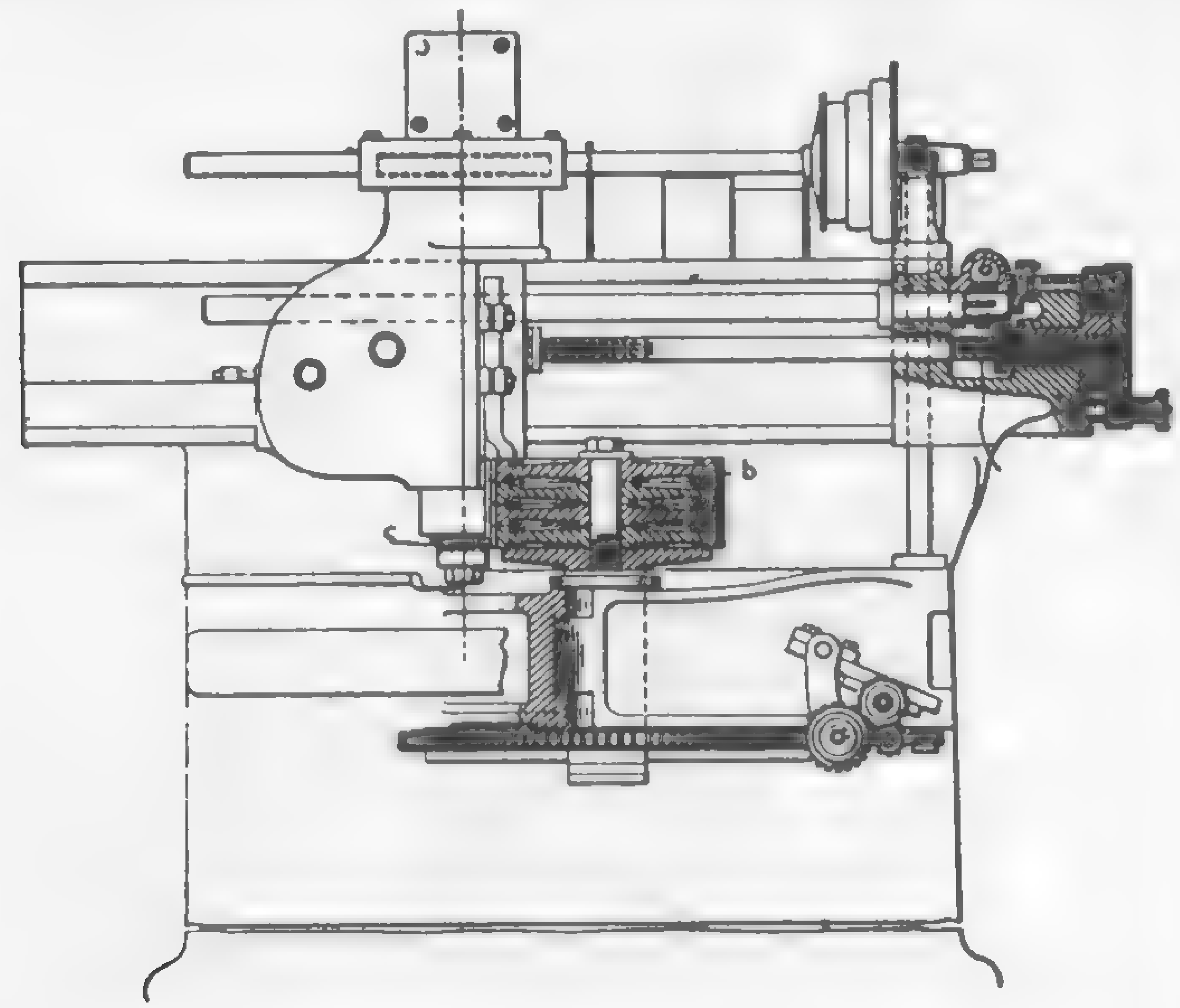
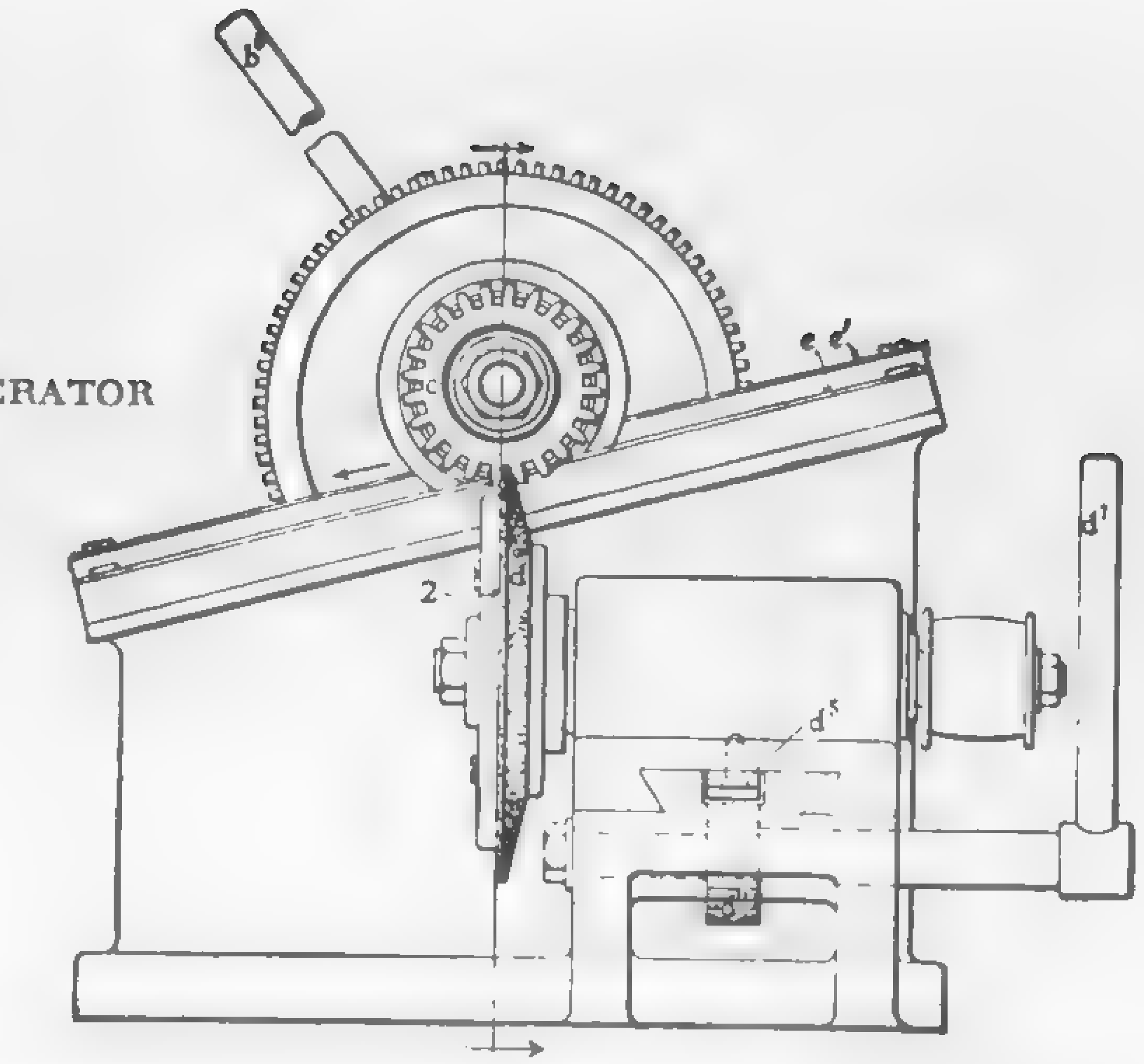


FIG. 40A. THE FELLOWS GEAR SHAPER (*American Machinist*)

The gear cutter works on a specialized slotter and is reciprocated vertically. On internal gears it planes downwards; on spur gears it may plane on either stroke, but originally planed on the up stroke. Both the cutter and the gear blank are slowly rotated in mesh.

The gear cutter is made with the addenda of its cutting teeth greater than that of the teeth it is to cut, in order that the one cutter may cut all sizes and the rack for gears of a given pitch; and there will be adequate space at the bottoms of the teeth to that all will mesh together properly. This same tool can, of course, be used to cut internal gears, but bevel

FIG. 40B.
THE FELLOWS CUTTER
AND BEVEL-GEAR GENERATOR
(*American Machinist*)

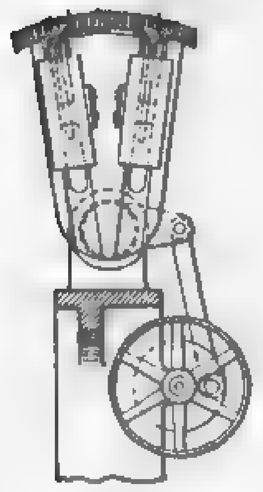


gears cannot be cut on this machine. In fact it is important that the axis of the path of the generating gear cutter coincide exactly with the axis of the gear blank.

The crux of Fellows' work is the method of grinding the hardened gear cutters. These are first cut, a little large, by another hardened gear cutter and then put in the cutter-grinding machine. This machine can finish cutters for either straight or helical spur gears. It is shown in Fig. 40b in the hand-operated form which was later made automatic. The emery wheel "*d*" has one side "2" a plane surface; the other side of the wheel is tapered as necessary to clear the teeth being generated. The flat side of the emery wheel acts on the tooth side of the blank "*C*" like one side of an involute rack tooth. The blank is rolled in mesh with it by means of the handle "b^5". Two flexible steel bands "*e*" and "e'" provide the movement of translation. For a large blank the emery wheel may be moved across its face by the rack "d^5", pinion "d^6" and handle "d^7". This mechanism is also used to withdraw the wheel when indexing the blank. The angle corresponding to that of the involute rack tooth is provided by having the blank carrier at an angle to the horizontal.

Fellows also developed on these same principles a rack-generating machine and one for helical gears, as well as a worm-gear shaper.

A machine designed by H. C. Warren especially for making bevel gears for chainless bicycles was in extensive use by the Pope Company of Hartford, Connecticut.[53] As shown in Fig. 41, these machines had two rotary[54] cutters mounted on axes at an angle with each other. These cutters were beveled to give in effect opposite sides of adjacent teeth in a crown gear. The blank rotates on its axis. An oscillating carrier on which are slidably mounted the cutter carriages is swung around the axis of the imaginary crown gear to give to the cutters and the blank the necessary relative movements.



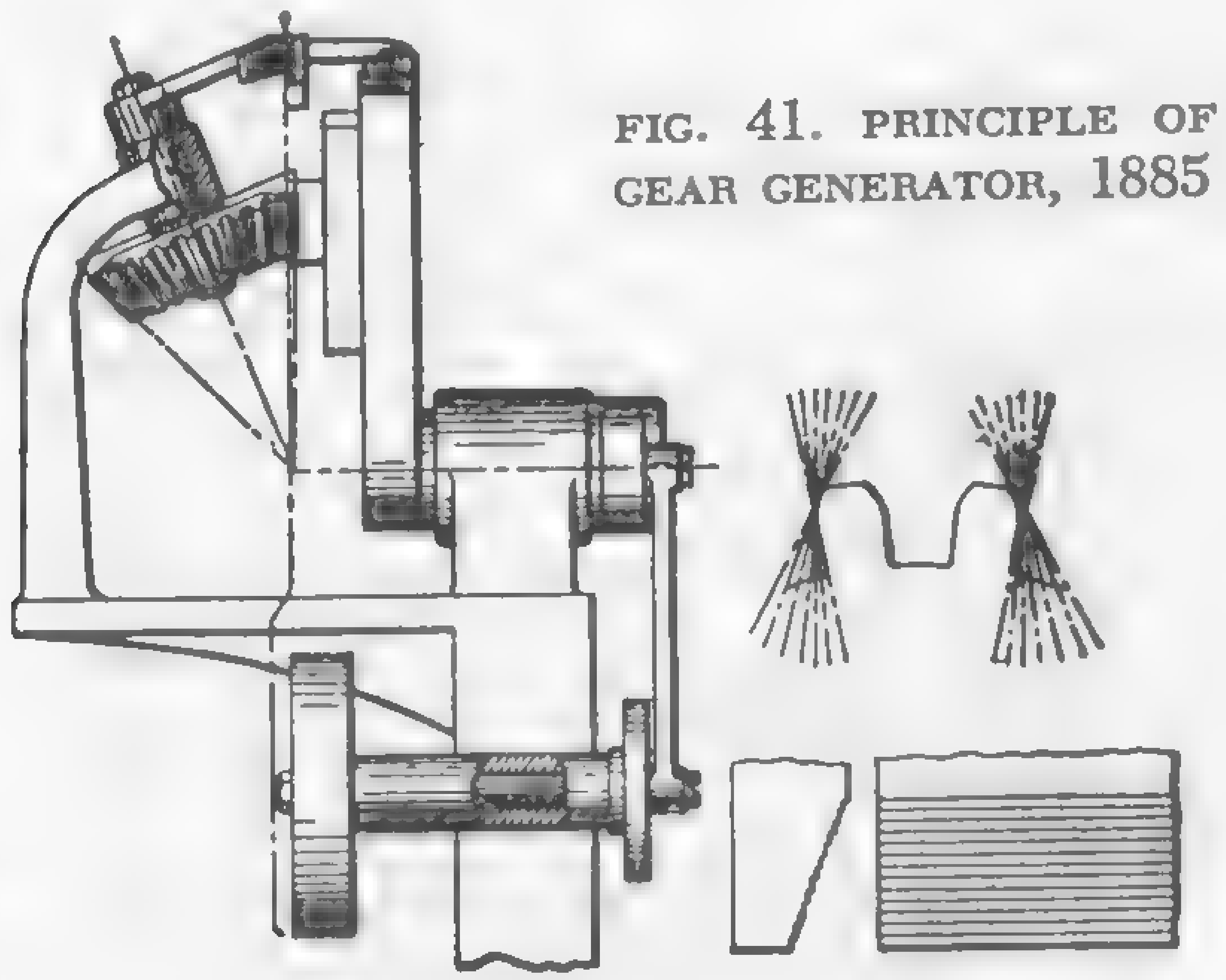
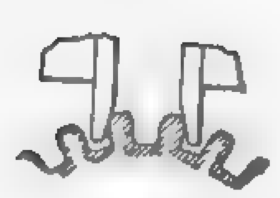


FIG. 41. PRINCIPLE OF THE WARREN BEVEL-GEAR GENERATOR, 1885 (*American Machinist*)



In 1898 James E. Gleason[55] invented a bevel-gear generating machine (Fig. 42). This had a rotary cutter which finished both sides of a space in a single traverse across the face of the blank. To do this the rolling or intermeshing movement is obtained by rotating the blank and swinging the cutter carrier about the vertical axis through the cone apex of the blank. A lateral movement of the cutter is also provided so that first one side and then the other is in the

53. *Am. Mach.*, Mar. 24, 1898, p. 211.
54. As shown in Fig. 41, the cutters are broaches or files and do not rotate, and the feed motion is not shown since it is not required for these cutters.
55. See his Patent No. 605,249 of June 7, 1898, and Fred J. Miller, "Bevel Gear Cutting Machines at Paris" [1900], in *Trans. A.S.M.E.*, 1901, pp. 700-712.

FIG. 42. GLEASON BEVEL-GEAR GENERATOR, 1898 (*American Machinist*)

vertical plane through the axis about which the blank is rolling. These motions are coordinated so that each time the cutter is moved laterally the direction of oscillation of the blank and of the cutter is changed. This takes place after every two revolutions of the cutter.

The Gleason machine is of interest to theory for it is the obverse of the Bilgram machine in its operation. Bilgram held his imaginary rack fixed endwise and rolled the gear blank past it as a gear would roll in a rack. Gleason's machine had the gear blank turning on a fixed center, and the rack travels endwise with the feed. The resulting *relative* motion is, of course, the same. Gleason's machine was, however, fully automatic from the beginning.

The principle of the machine of C. D. Rice[56] is of interest only because it combines a kind of generating mechanism with a master gear used as a templet.

After the Fellows Gear Shaper the molding-generating machine which had the greatest commercial importance was the Brown & Sharpe machine, designed in 1900 by Oscar Beale (1843-1911).[57] In this machine both sides of a space are finished at the same time. This is done by using two

56. *Am. Mach.*, May 10, 1900, p. 440, and Fred J. Miller, *op. cit.*, pp. 712-719.
57. *Am. Mach.*, Jan. 29, 1903, pp. 145-150, and 1912, p. 1027.

toothed discs mounted on inclined axes with the teeth of one occupying the spaces of the other. The outer cutting faces of the discs act as the two sides of an involute worm-gear tooth. There is no motion of the axes of the cutters; all other motions—rotation and translation—are given to the blank. Provision is made for taking a finishing cut as well as to form bevel gears of various diameters, pitch and pressure angle (Fig. 43).

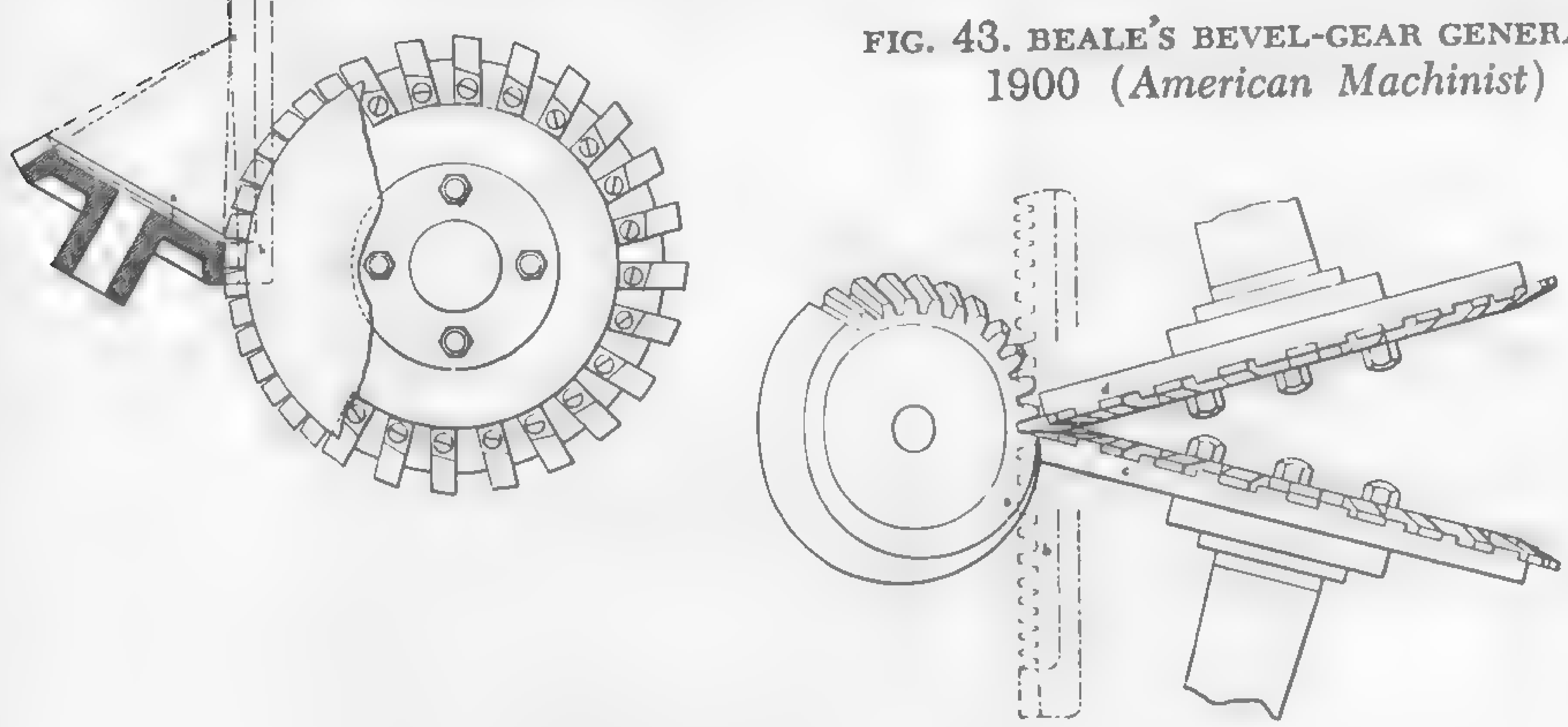


FIG. 43. BEALE'S BEVEL-GEAR GENERATOR, 1900 (*American Machinist*)

The appearance of the automobile produced a much greater demand for types of gears seldom used before 1900, and this was reflected in gear-cutting machines designed to cut helical gears, both spur and bevel, as well as skew-bevel gears. Machines utilizing the generating principle were produced to meet this demand, but the hobbing method was soon to dominate this field.

The French machine of Monneret[58] is an intricate but ingenious way of generating helical bevel-gear teeth. In it the tool is reciprocated in a plane representing the side of a crown-gear tooth. The tool holder slides on a guideway pivoted on an axis coinciding with that of the imaginary crown

58. *Am. Mach.*, July 19, 1900, p. 683, and Fred J. Miller *op. cit.*, pp. 686-695.

gear. At the same time the helical groove is generated by a slight rotation of the blank, which also serves to index the blank for the next cut.

As was mentioned in the introduction, the process of cutting a helix or a thread in a lathe is fundamentally the same as that of the special case of cutting a spiral gear or a worm by the formed-tool method. This can be seen very clearly in the machine for generating spiral gears described in the 1889 patent of E. P. and H. C. Walter shown in Fig. 44. It has a tool *O* in the form of an involute rack tooth whose only motion is a reciprocation by a screw and reversing pulley arrangement as shown. The blank is fed along the direction of its axis and also rotated by a set of change gears and a worm-and-pinion mechanism.

It will be seen that this is the same process as cutting a thread in a lathe, except that the tool is given an additional cutting movement. The same relative movement takes place whether, as in a lathe, the tool is moved parallel to the axis of the rotating work; or whether, as here, the rotating work is moved past the tool. This in effect produces a motion sometimes used in grinding machines in which, in effect, the

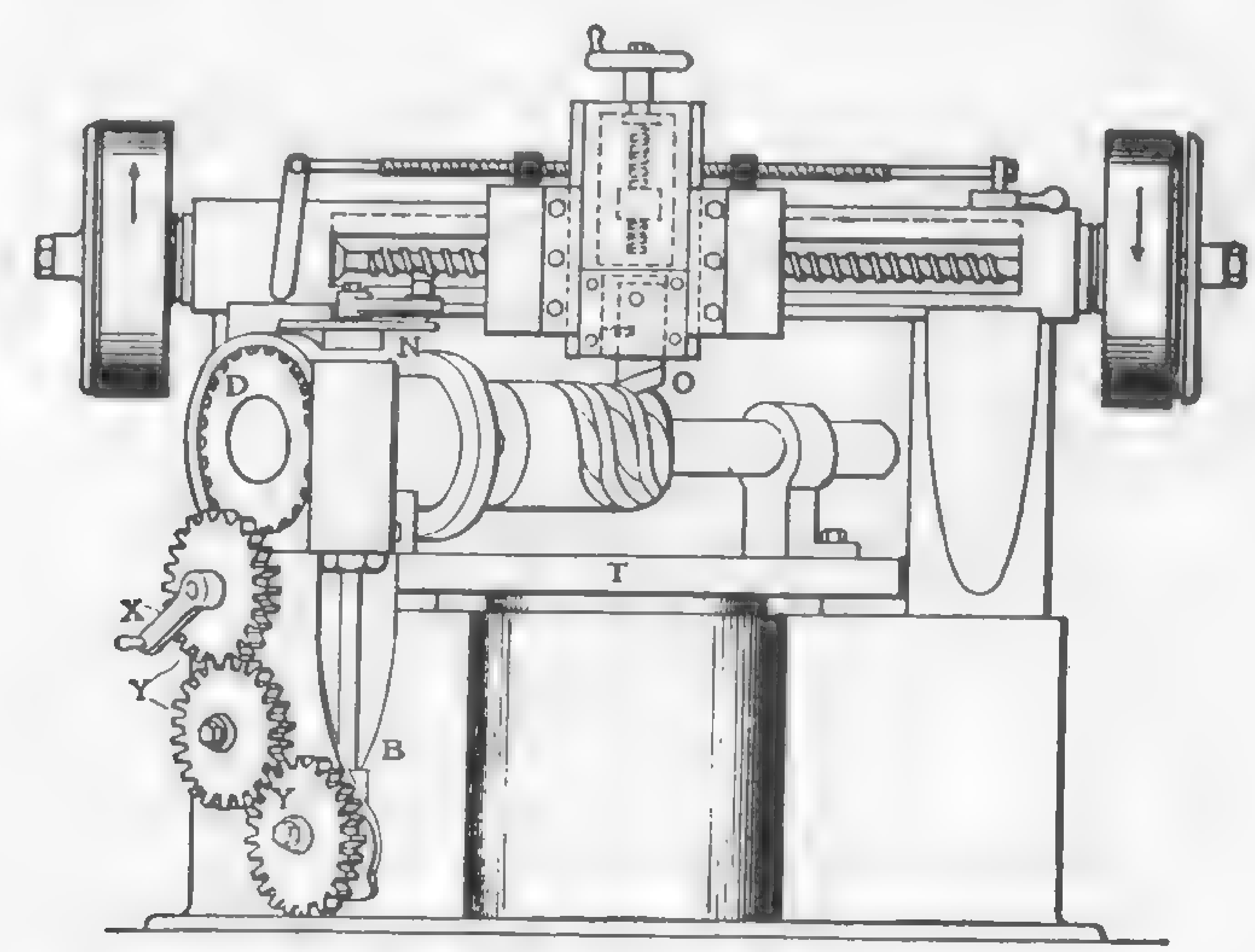


FIG. 44. WALTER'S SPIRAL-GEAR GENERATING MACHINE, 1889 (*American Machinist*)

lathe's headstock and tailstock are connected together and fed past a tool carriage clamped to the bed. In the screw-cutting lathe the change gears move the tool; in the Walter machine the ratio between the rotation of the work and the travel of the tool parallel to it is fixed by having the change gears drive the work.

If in a lathe we cut a very coarse-pitch thread (as we do in a worm or a spiral gear), the lead screw has to do too much of the work of the cutting. In the Walter machine this is avoided by giving the cutting tool an additional movement—here, reciprocating but in later machines, rotary. This is seen in the thread-milling machine—of which a hobbing gear-cutting machine generating worm or spiral gears is but the general type, and the hobbing machine is but a special type of molding-generating machine. A screw thread is nothing but a worm; the teeth of spiral gears are only small sections of screw threads of coarse pitch; and the ordinary spur gear is only a spiral gear whose helix angle is equal to zero.

As the dates of these inventions indicate, after 1884 the describing-generating method has come less and less to be used and the molding-generating method more widespread, especially in the form of the hobbing gear-cutting machine so convenient for making gears for automobiles, turbines, and other machinery requiring high speed and quiet running under substantial loads.

HOBBING TYPES

Most of the methods of gear cutting that we have considered thus far had in common one serious defect under production conditions. As the demand for gears increased the tendency was, of course, to speed up existing machines. At higher cutting rates a new problem appeared—local heating, with resulting local deformation of the gear blank near the area being cut, and also excessive wear on the cutting edge. In order to avoid this deformation it became customary to do one tooth, then skip several, and so on all around the blank,

until the gear was completed, which spread the heating more uniformly through the blank. This local heating was caused by the fact that only one tooth at a time was being cut. The only exceptions were the Fellows Gear Shaper, which used a hardened gear as a cutter, both the gear and the blank rotating, so that several teeth were being cut at once and the teeth of the cutter actually in use changed constantly. Another machine of some significance in practice, the Sunderland,[59] did use, rather than a single rack tooth, an actual hardened rack cutter, in length greater than the circumference of the gear to be cut. This gave, then, more than one tooth being cut at a time and also constant change of the working cutting edge. However, these two types required a reciprocating planer-like motion of the cutter. The Fellows gear shaper had, of course, many other advantages.

But it had been recognized that the worm is a form of continuous rack and that all that was necessary to cut gears with it was to provide cutting edges on it—to make a hob (Fig. 45). Teeth had been cut by this method probably for the first time by Ramsden in 1768.[60] But this was only instrument work. The first application of the hobbing principle to a production machine was made in 1835 by Joseph Whitworth[61] but his machine would cut only spiral gears. In 1839 Pfaff had used a hob to cut worm gears, as had Whitehead in 1853. But in 1856 Christian Schiele took out a patent[62] for a machine intended to do thread milling; as we have seen, this is in theory the same as cutting teeth on gears. Schiele's patent specifications provided for gearing the blank to the hob in the proper ratio. This was the first hobbing machine to do this. Schiele used his hob at 90° to cut spur and helical gears, and also cut worm gears on his machine. Although Schiele's machine is clearly the beginning of the gear-hobbing type, we do not know if it ever appeared in practice.

59. See *Am. Mach.*, Mar. 10, 1910, pp. 443-445, for a detailed description.
60. See above p. 26.
61. See his British Patent No. 6850 of 11 June 1835; also see *Am. Mach.*, Aug. 4, 1888, p. 5.
62. British Patent No. 2896 of 6 Dec. 1856.

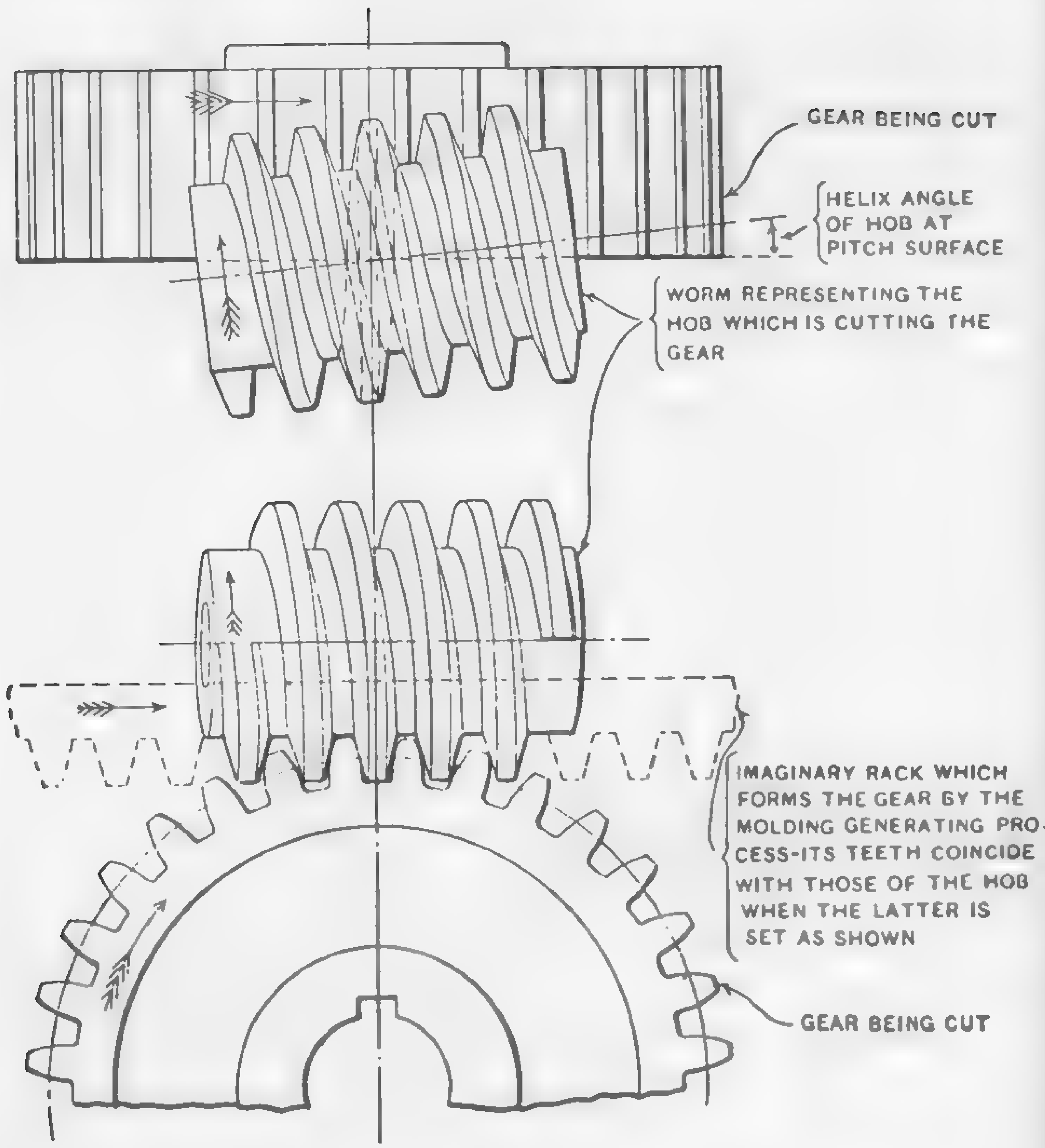


FIG. 45. THE PRINCIPLE OF THE HOBBING PROCESS OF FORMING SPUR GEARS (*Flanders*) (*Note that hobbing is a milling method applied to the molding-generating principle of Fig.* 11F.)

Another hobbing machine of which we have only the patent was that of Henry Belfield in 1871.[63] This was an attachment for a lathe. The cutter was carried and driven by the lathe centers. The blank was pivoted on the slide rest, and fed by the threads of the cutter without any gearing.

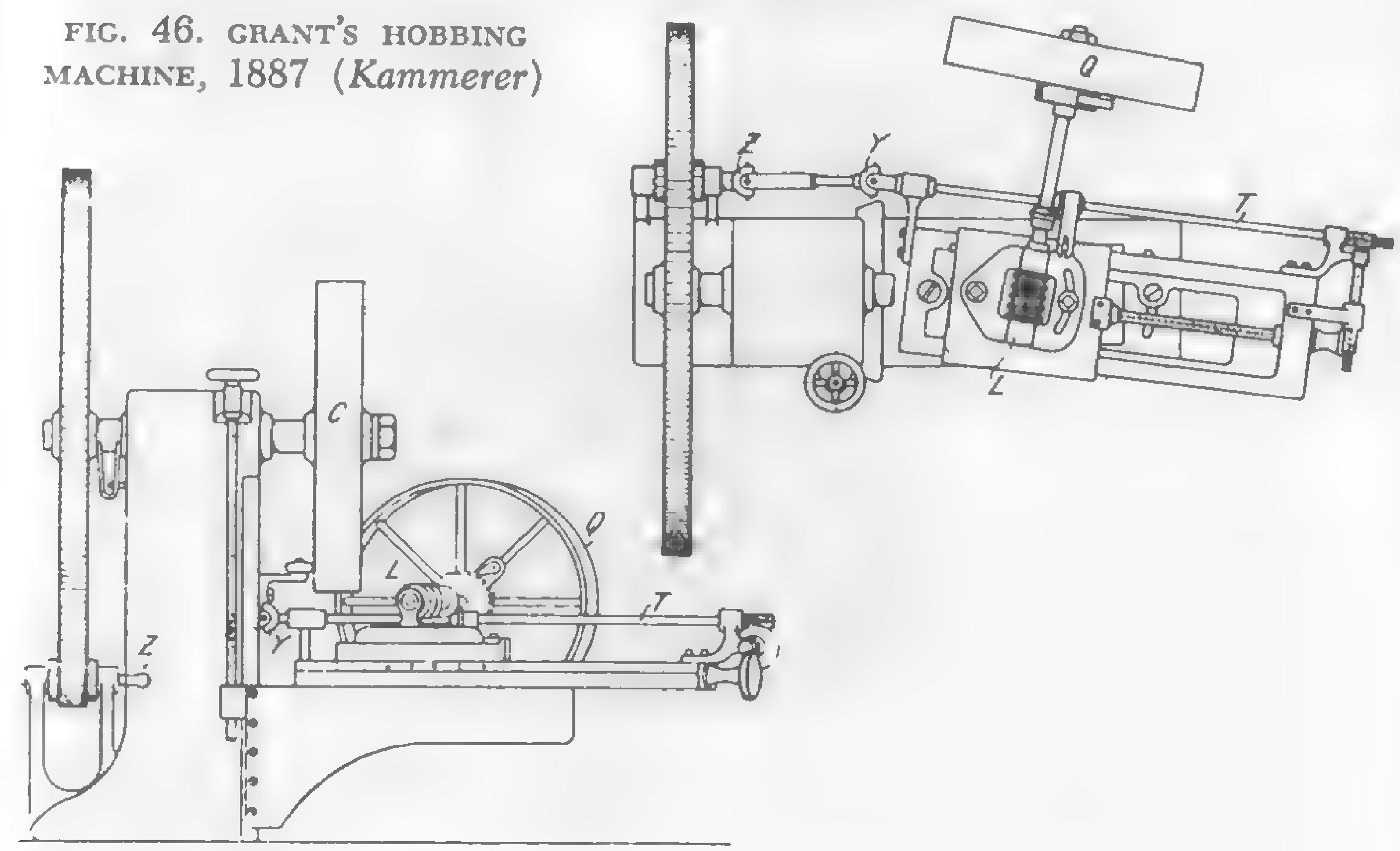
The first machine that we know was actually in use is that of George B. Grant.[64] As shown in Fig. 46 it was adapted "for cutting either straight or spiral teeth of spur gears; and

63. See his Patent No. 120,023 of Oct. 17, 1871.
64. See his Patent No. 405,030 of June 11, 1889, first application Feb. 18, 1887; also *Am. Mach.*, 2 July 1908, p. 15.

it consists in certain novel mechanisms to feed a spiral cutting tool or hob across the face of the gear blank being cut while both gear blank and cutter are revolving together." Grant correctly points out the principal advantages of the hobbing process in speed and accuracy as well as convenience.

The slow development from Whitworth and Schiele's gear-hobbing machines until the Juengst machine of 1893 and the Reinecker of 1894 can be explained satisfactorily if one looks carefully at the actual technical problem. First, the demands for rapid production of gears did not appear until the 1860s. This meant higher speeds of operation of existing machines, plus the appearance of automatic gear cutting machines late in that decade. Both of these factors resulted in the local heating problem referred to above. But there was another problem. Until the wide acceptance of involute teeth the difficulties of cutting a hob for epicycloidal teeth would have made this seem hardly a promising solution. The involute hob, like the involute rack, has substantially straight sides, which are easily made. It also took a little time for the "scientific mechanics" to realize that only one

FIG. 46. GRANT'S HOBBING MACHINE, 1887 (*Kammerer*)



hob is required for all gears of the same pitch. There were also problems of precision set up when using the hobbing method in that one tooth must be set exactly on the center line of the machine. And the axis of the hob must be set at an exact angle to the axis of the gear equal to the angle of spiral of the thread of the hob.

There were also difficulties in making the hobs. Their hardening produced some distortion; and the hardened hob had to be ground to an exact final form—not an easy process in the then state of the grinding machine. In addition the hobbing method was believed to cut little flats on the gear teeth. Nonetheless, if the technical problems could be overcome—and they were— the advantages far outweighed the disadvantages.

The Juengst[65] and the Reinecker hobbing machines are shown in Figs. 47 and 48. In both these early machines the gear-blank spindle is horizontal and the hobbing cutter is fed in horizontally. Since that time most gear hobbers have the blank on a vertical spindle and the cutter is fed vertically downwards.

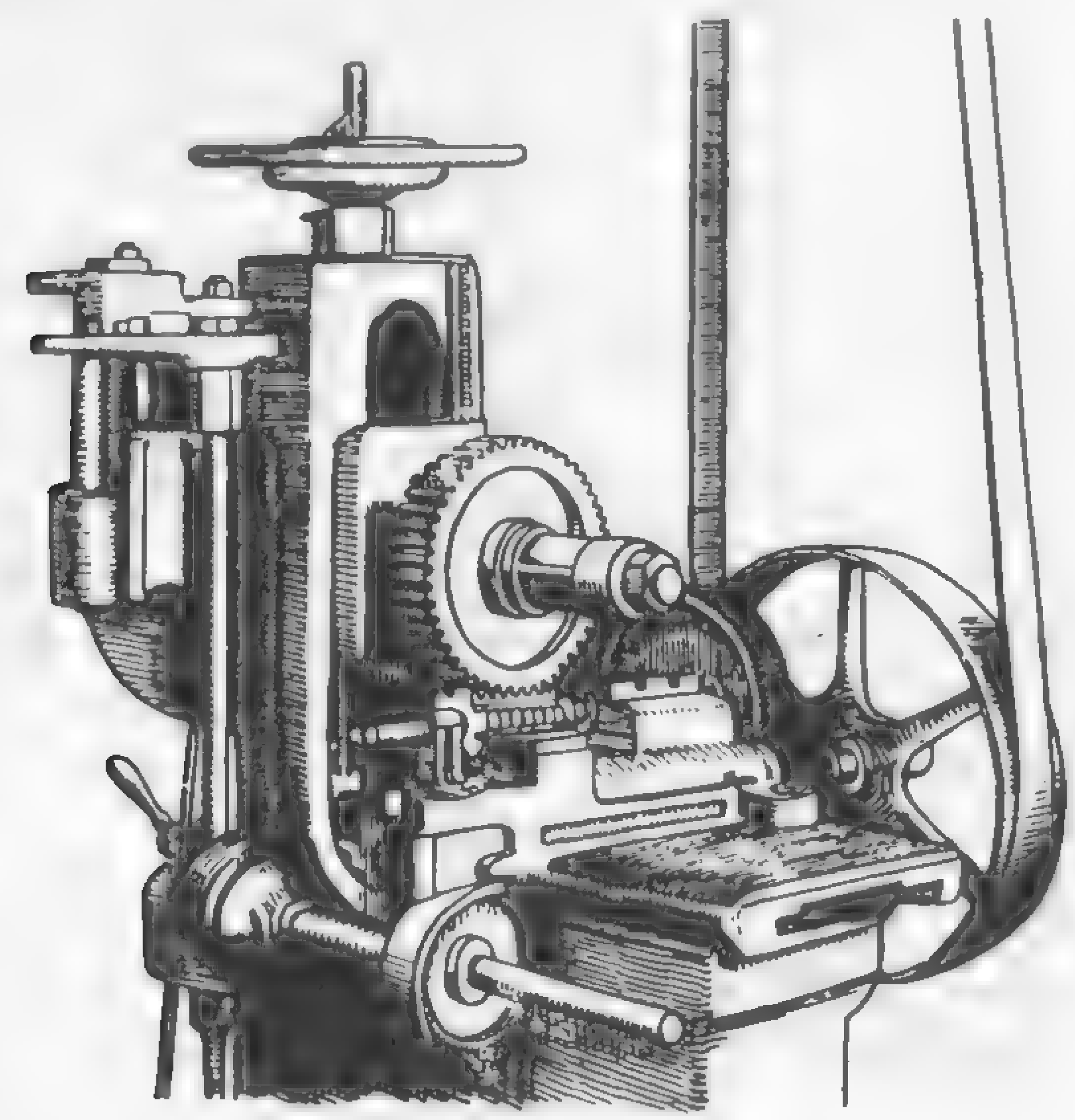


FIG. 47. JUENGST GEAR-HOBBING MACHINE, 1893 (*Inst. Mech. Eng.*)

65. See Flanders, pp. 174-175.

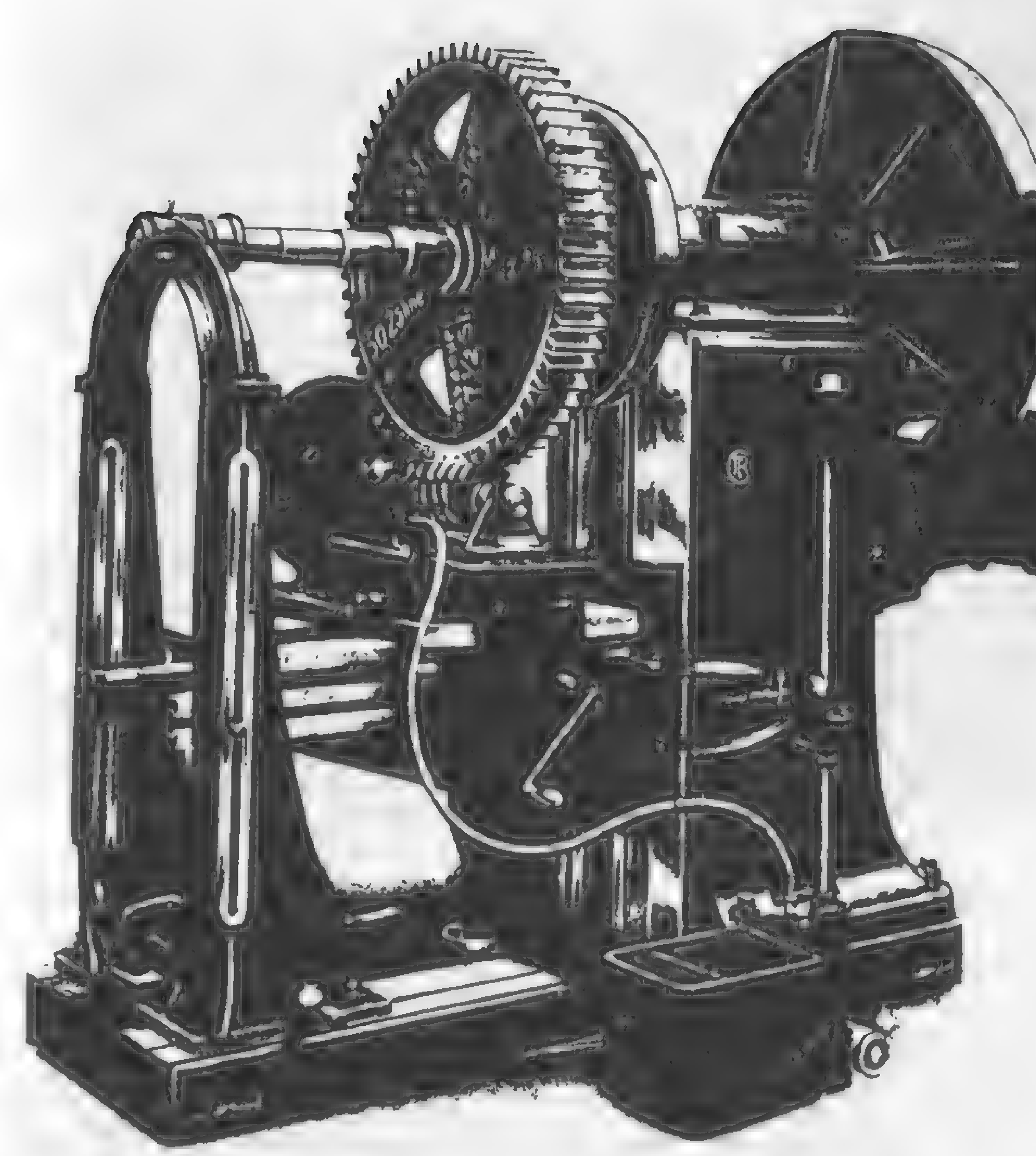


FIG. 48. REINECKER SPUR-GEAR HOBBING MACHINE, 1894 (*Inst. Mech. Eng.*)

The Pfauter machine[66] is shown in Fig. 49. This machine (*ca.* 1900) produced the change to the tangent cutter movement in which the cutter axis is no longer at 90° to the gear axis. Together with the simultaneous drive of Schiele it produced the basic elements of the hobbing gear-cutting machine. These machines were made in various sizes, the largest of which have now practically supplanted the templet-type machine for cutting large gears. Arrangements were provided for the hob to work at any angle, and the machine could cut

66. K. Kutzbach, "Vom Wesen und Werden des Pfauterverfahrens," in *Z.V.D.I.*, 1927, pp. 73-80. See Pfauter's German Patent No. 112,082 of 2 Sept. 1897.

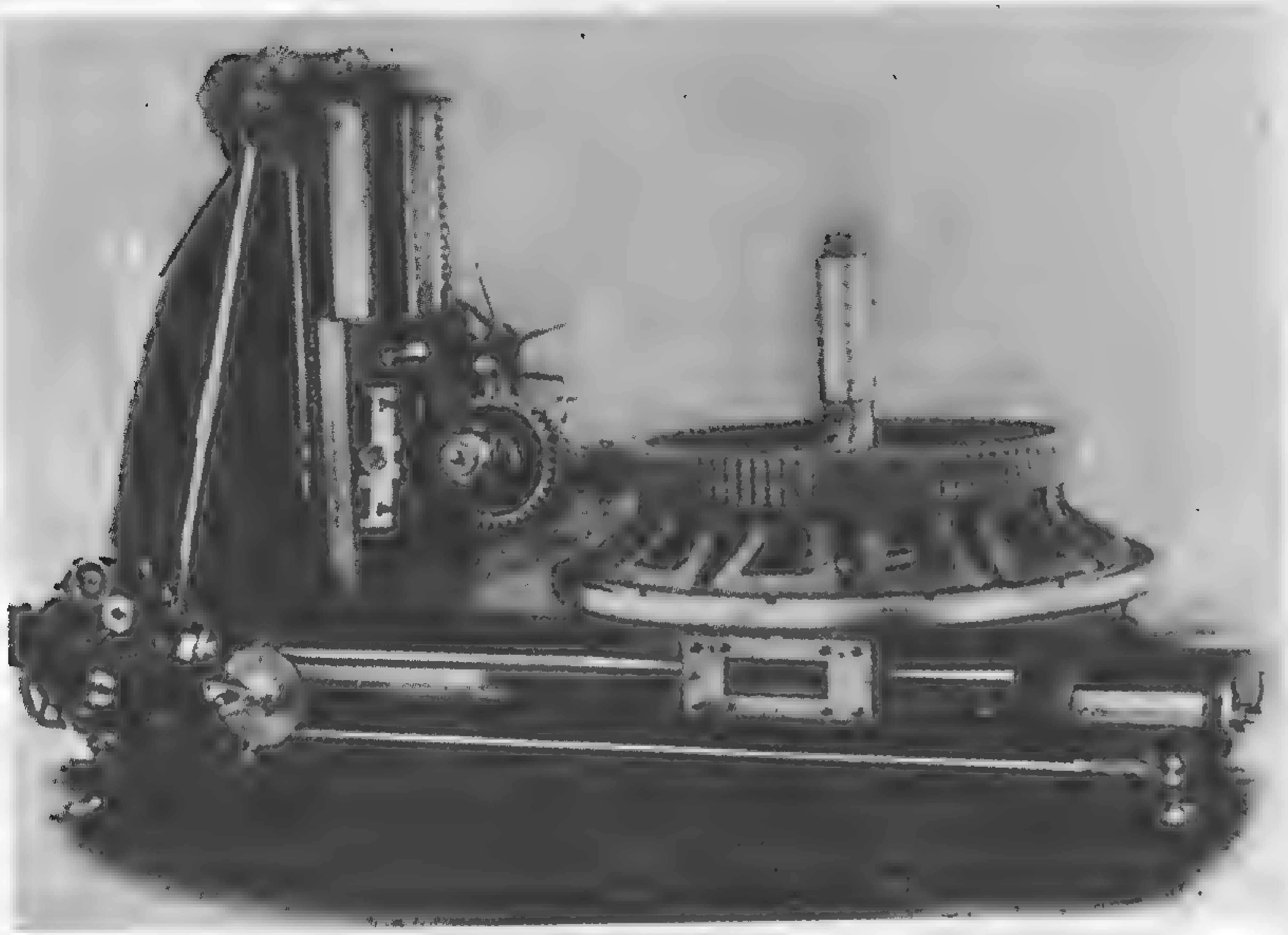


FIG. 49. PFAUTER GEAR-HOBBING MACHINE, 1897 (*Inst. Mech. Eng.*)

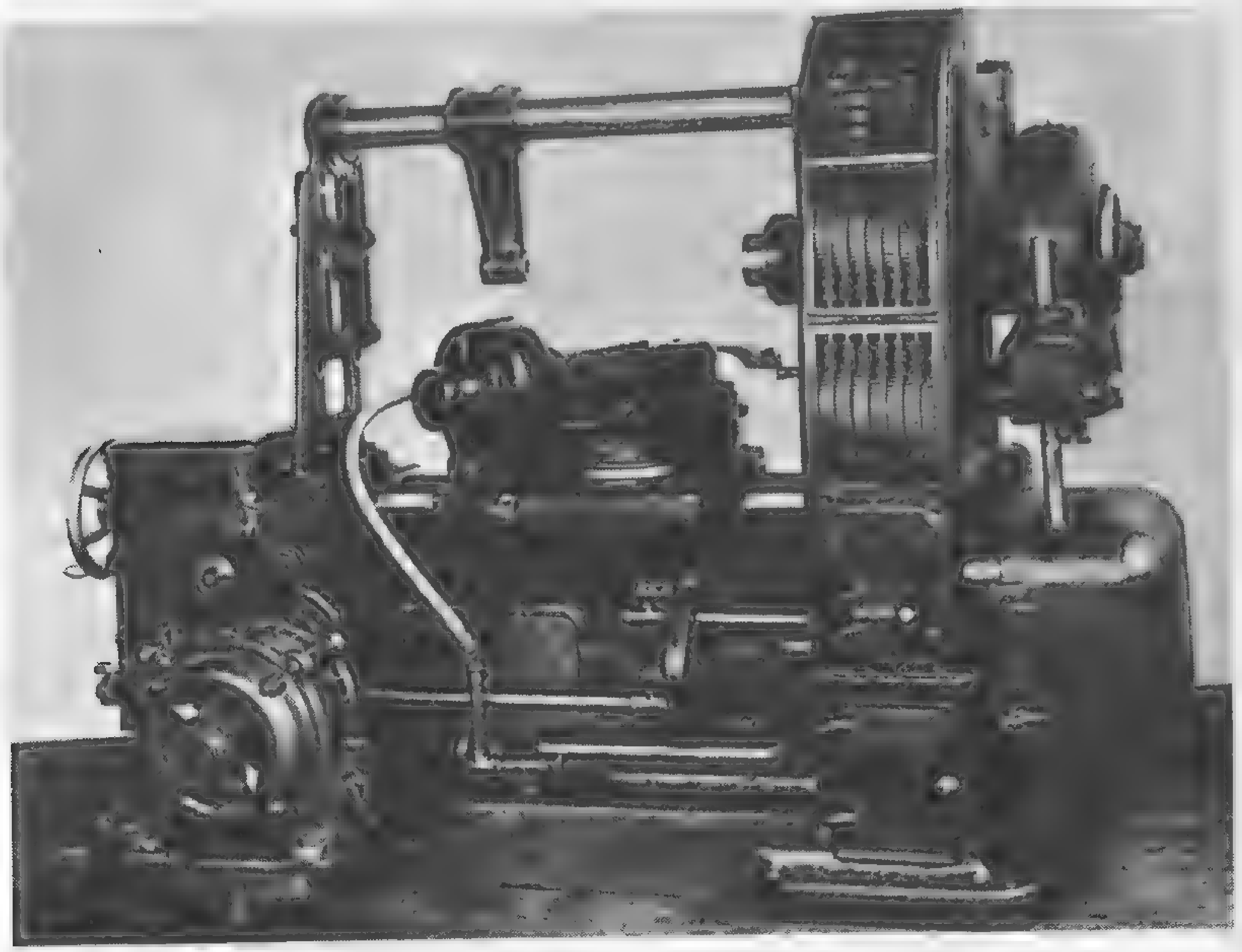


FIG. 50. BROWN & SHARPE GEAR HOBBER, 1915 (*Brown & Sharpe*)

spiral gears by a differential gearing which advanced or retarded the wormshaft by the amount desired.

Although the work of Pfauter established the basic principles of the gear-hobbing machine, actual manufacture of this type of machine gave rise to a number of problems not otherwise foreseen. A representative story is found in the files of Brown & Sharpe. We find that in early 1904 R. T. Wingo and E. H. Parks of their staff were sent to visit the De Laval Steam Turbine Company, the Newton Machine Tool Works and the plant of Hugo Bilgram, all of whom had had some experience in "the cutting of spur, spiral and worm gears with a hob."[67] Four principal points seemed important: (a) Difficulty of getting correct angles and alignment of the axis and center of the hob with the axis and center of the gear blank. (b) Accuracy of the manufacture of the hob was essential. This meant grinding it after hardening. The hob clearance needed to be ample for free working. (c) Hobbing clearly permitted much greater rates of removal of metal with equally good gears resulting. (d) But the stresses and load were such that much heavier machine construction was required.

Brown & Sharpe promptly went ahead on their own development of the hobbing method, for we find a report of about a year later.[68] This indicated that good quality gears could be made at a considerable saving in time by this method if the above conditions were strictly fulfilled. Evidently not knowing of the possibilities of the "barrel hob," they reject the method for internal gears. They also doubt its feasibility for cutting racks or the "quill gear."

The work was continued and the possibilities of hobs with inserted teeth considered[69] as well as Humpage's suggestion[70] for finishing gear teeth by grinding with a corun-

67. See their typescript report of Feb. 22, 1904, in Brown and Sharpe files.
68. Report of R. T. Wingo and O. J. Beale dated Mar. 16, 1905, typescript in Brown and Sharpe files.
69. As suggested in *Am. Mach.*, Jan. 28, 1909, p. 126.
70. *Trans. Inst. Mech. Eng.*, 1908, pp. 674, 691; and *Machinery*, Feb. 1909, p. 423. Automobile manufacturers had so much trouble in making an accurately finished hob, that their practice was to use hobbing only to rough

dum worm.[71] At any rate Brown & Sharpe went ahead and produced in 1915[72] a successful gear-hobbing machine, their No. 34 and No. 44 (Fig. 50).

By 1909 Flanders could describe twenty-four manufacturers of hobbing-type gear-cutting machines, all designed since 1900 to meet the demands of the automobile and other trades for spur, internal, rack, helical, and worm gears. The steam turbine brought in special gearing problems to meet high loads and extremely high speeds, and the solution was found in the herringbone type of gear. Fig. 51 shows the principle used by De Laval[73] to get the necessary exact indexing. The Wüst[74] type of gear obviated the necessity for making the herringbone in two parts. Various other types of gears caused for their manufacture a revival of White's use of the end mill to get the necessary forms.[75]

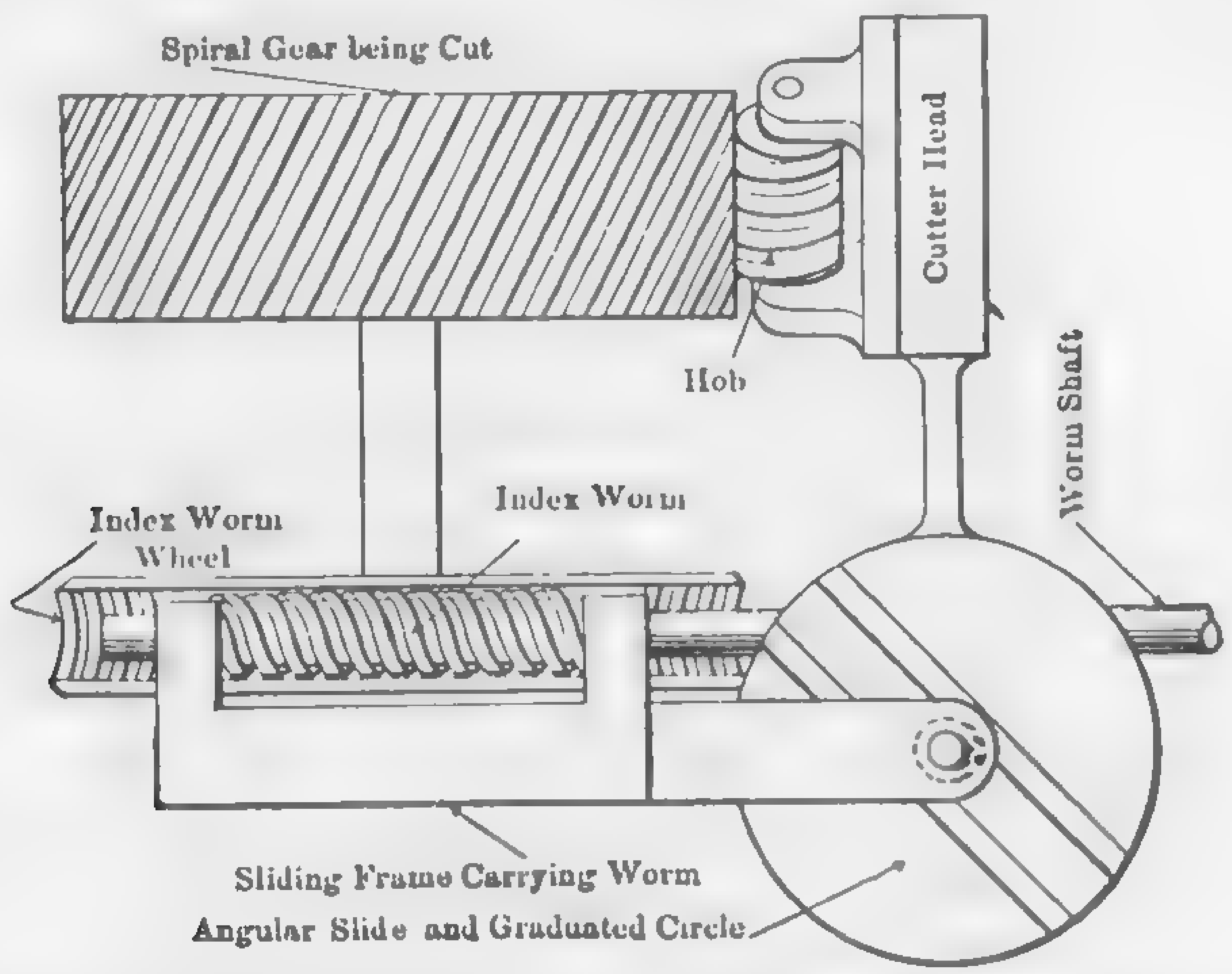


FIG. 51. DIAGRAM SHOWING SPIRAL MECHANISM IN DE LAVAL MACHINE (*Flanders*)

out the teeth and get the advantages of perfect spacing and high rate of metal removal. The teeth were then finished with milling cutters.

71. MS of R. T. Wingo dated Mar. 2, 1909, in Brown and Sharpe files.
72. See their catalogue for 1925, pp. 192-195.
73. See Flanders, pp. 176-177.
74. See Flanders, p. 178.
75. For the later history of gear hobbing and a most interesting general analysis of the methods of gear cutting see Kutzbach, *Z.V.D.I., loc. cit.*, 1928.

Of the few gear-manufacturing gear machines which use the impression method we have had two types—one actually forming the metal gear, the other forming the sand mold for casting gears.

A kind of impression gear machine had been used by Continental watchmakers in the 18th century. It was called a "deepening tool" and was used to adjust the engagement of wheels after they had been cut.[76]

In 1899 Brown & Sharpe put on the market a more elaborate machine, designed by Oscar J. Beale for finishing and correcting bevel gears by impression rather than by cutting or grinding. Like the deepening tool, it did not, of course, cut the gear directly from the blank.[77]

This was a modification, on a production basis, of the Ingold device.[78] This Swiss device for finishing watch wheels was invented by Pierre Ingold of Biel. It used a tool like a spur gear, but having file surfaces on its teeth. This cutter was run in mesh with already cut watch wheels to finish them. It required, of course, a different cutter for each pitch, and the cutters were difficult and expensive to make.

As shown in Fig. 52 Beale's machine used a hardened crown gear which was run in mesh with the bevel gear being formed. No additional cutting movement is required. This forming gear was made by cutting down the straight-sided teeth of the appropriate crown gear to various heights, leaving sharp edges. This hardened gear is brought up to mesh with the gear to be finished and burnishes and compresses the teeth of the bevel gear to the correct shape. It is then finished with an Ingold cutter.

The only early machine utilizing a full-impression method was that of H. N. Anderson. His machine rolled a gear

76. See Rees, *Encyclopedia*, Phil., 1806-?, Article "Clock Tools" and "Horology," Plate XXI, Fig. 6. This device can also be found in Berthoud, *op. cit.*, Chap. 25, and Plate IV.
77. See Flanders, pp. 275-278, and *Am. Mach.*, Apr. 6, 1899, p. 272.
78. *Am. Mach.*, Apr. 27, 1899, p. 364.

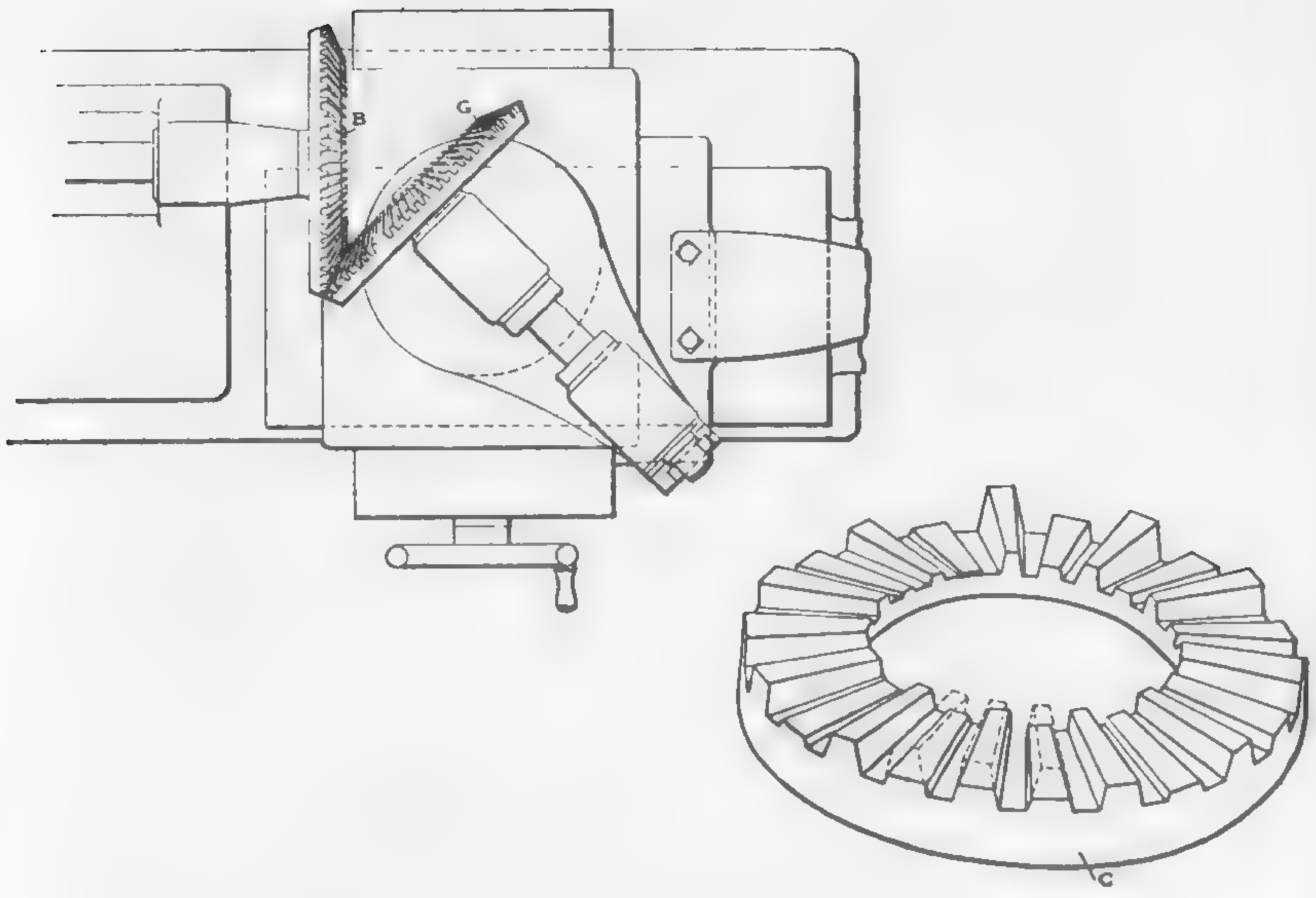


FIG. 52. BEALE'S IMPRESSION MACHINE, 1899
(*American Machinist*)

blank, heated to a forging temperature, against an accurately cut toothed roller. It was claimed that this method produced a stronger and longer wearing gear.[79]

In 1951 Ernst and Benjamin Grob developed a successful method for cold rolling gears.[80] It is based upon an extension of the highly developed techniques of accurate thread rolling, and can produce spur, bevel, and helical gears.[81]

Because of the warping and the cost of the large numbers of wooden patterns required for cast gears, impression-type machines have also been used in making the molds for

79. See *Machinery*, Nov. 1910, pp. 235-237, and *Am. Mach.*, Aug. 20, 1914, p. 348. Anderson's basic principle was anticipated in John Comly's patent No. 132,899 of Nov. 12, 1872, but Comly was far from having worked out a practical device.
80. Patent No. 2,715,846 of Aug. 23, 1955. Flied May 16, 1951.
81. Letter from Benjamin Grob, Jan. 22, 1958. The theoretical basis of this method is indicated on pp. 103-104. The development of rolling screw threads will be included in a later monograph on the History of the Automatic Screw Machine.

the very large cast gears. Lohse[82] says that the first molding machine goes back to 1826 in France with Sonolet. However, the first true gear-molding machine was patented by J. G. Hoffman in Prussia on 11 October 1839. His design was the prototype of the gear-molding machine. In France gear-molding machines were developed by Ferrouilh in 1852 and de Louvrie prior to 1857. Quite independently P. R. Jackson[83] adapted the ordinary vertical wheel-cutting engine to gear molding by adding a turntable upon which the sand mold was formed. He also made provision for casting bevel gears, added a dividing plate and gearing and made improvements in the form of the gear-segment pattern. Jackson's machine, the first to be constructed, was shown at the London Exhibition of 1851 (Fig. 53).

Within the next ten years similar machines were in use in France. Hoffman's machine was not actually constructed

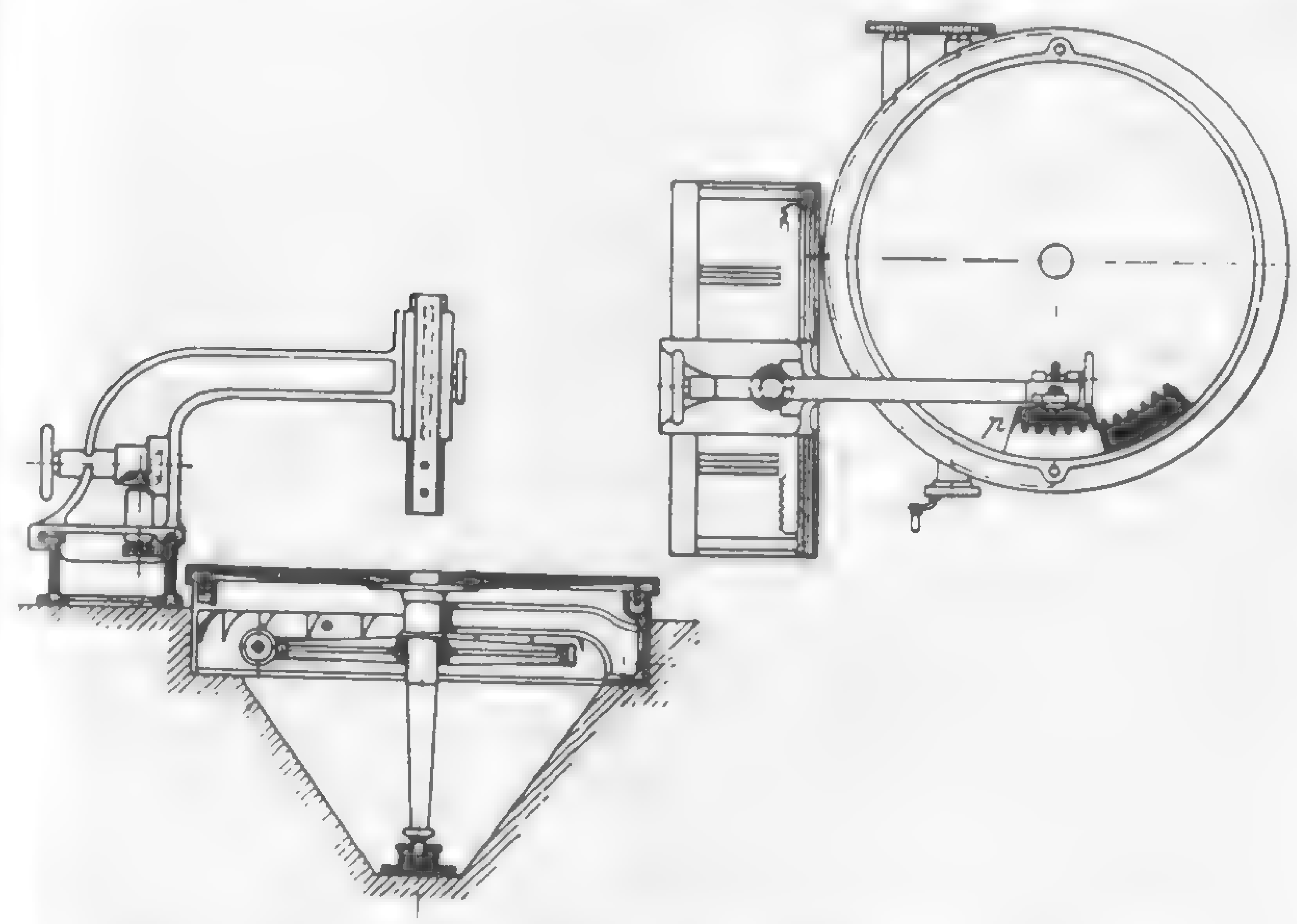


FIG. 53. JACKSON'S GEAR-MOLDING MACHINE, 1844 (*Lohse*)

82. U. Lohse, "Die geschichtliche Entwicklung der Eisengiesserei seit Beginn des 19 Jahrhunderts," *B.G.T.I.*, Vol. II, 1910.
83. British Patent No. 10,073 (Old Law) of 24 Feb. 1844, and his later patent No. 2261 of 3 Oct. 1853 (Fig. 5). The dates given for Jackson in Matschoss, and in O. Kammerer, *B.G.T.I.*, Bd. IV, 1912, are misleading.

until 1859, by the firm of Otto Gruson & Company of Magdeburg. An important development was that of Johann Renk of Augsburg[84] who placed the upright supporting the arm at the center, rotated it about its vertical axis by a worm gear, and used a fixed table on which to support the mold. A rather interesting combination of both Jackson's and Renk's principles was incorporated in the gear-molding machine of the Fulton Iron Works, St. Louis, Missouri.[85]

With the formed-tooth milling cutter, the templet machine, the generating and the hobbing machines, even the impression machines, the skill and accuracy required for production gears were all built into the gear-cutting machine. All that remained was to make it automatic.

Automatic Gear-Cutting Machines, 1875-1910

The first gear-cutting machine to be fully automatic, that is, from the initial contact of the tool with the gear blank to continue by itself until all the teeth are completed and then to stop, was that of Christopher Polhem described above (Fig. 6). In his first power-driven machine we find a fully automatic action provided for by various cams, ratchets, and gear trains. As in so many other aspects of Polhem's work, the generations after him had far less of his vision of the value of specialized, power-driven machinery, having the necessary skill built into the machine, and therefore able to use relatively unskilled labor for its operation. There is therefore a delay of a century and a quarter until we again find automatic gear-cutting machinery. The reasons for this delay are quite simple—there was no demand for machinery for a mass production organization such as Polhem envisaged, until the 1860's in the U.S.A. When the demand for cut metallic gears became, during and after our Civil War, sufficiently great, the lack of skilled machinists, the perfection of the hand-controlled gear-cutting machine (at least of the formed milling-cutter type), and the technical developments already made in other machinery, then created a situation to which the automatic gear-cutting machine was a spontaneous answer.

84. German Patent No. 90,716 of 21 Apr. 1896. Renk's principle had been anticipated in America. See *Am. Mach.*, Mar. 31, 1888, p. 7.
85. *Am. Mach*, 1901, pp. 524-527.

The first such machine of which we have any information was that built by Gage, Warner, and Whitney of Nashua, New Hampshire, about 1860. It was photographed still in operation in 1896. The description given[86] tells us something of how its automatic features were provided. It was of the formed milling-cutter type, the gear-blank spindle being vertical, and had automatic power feed into the tooth depth and vertically across the face of the tooth. It could cut spur gears only, but these as great as eight feet in diameter. The indexing device was rather unusual. The feed was geared to a shaft operating a gear segment engaged with a gear mounting a pivoted weight. The feed across the face of the tooth raised the weight. When this passed a center it fell on the other side. This action threw a clutch in the cutter feed to the opposite position, and the cutter returned to the starting position. Its return caused the gear blank to be then indexed for the next tooth, the ball fell over onto the original side, and the action proceeded to cut another tooth. Adequate change gears were provided for cutting different sizes and pitches of gears.

At the Paris Exhibition of 1867 William Sellers of Philadelphia displayed an automatic gear-cutting machine, the first modern one of which we have adequate details[87] (Fig. 54). It was of the by then conventional formed milling-cutter type. Its automatic feature used stops operated by a screw, on the slide carrying the cutter spindle, to actuate the feed and return movement. Therefore there was no motion of the cutter-spindle carriage during reversing or indexing. The same mechanism unlatched the indexing mech-

86. *Machinery*, Nov. 1896, pp. 83, 84.
87. *Engineering*, Vol. 3, p. 675. Also see his Patent No. 99,356 of Feb. 1, 1870.

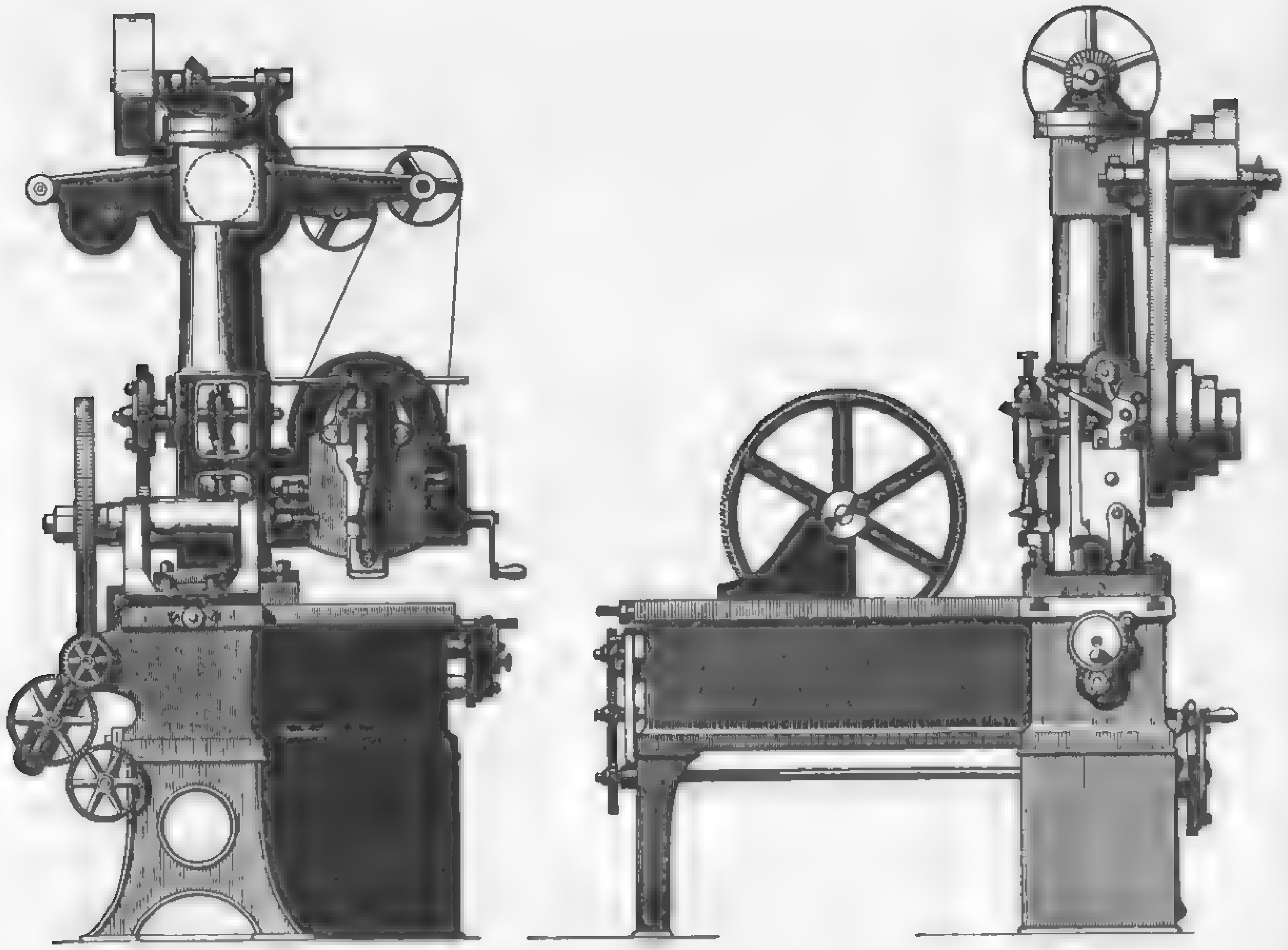


FIG. 54. SELLERS' AUTOMATIC GEAR CUTTER, 1867 (*Engineering*)

anism, operated the indexing to the next tooth, and then closed the indexing latch so that no indexing motion could take place while the cutter was engaged.

All motions in the cycle of this machine depended on the successful completion of the preceding one. Unless a fresh space was presented to the cutter for its work after each cycle, the machine would not operate; it therefore stopped when each gear was completed.

The year 1877 was an important one for the automatic gear-cutting machine, for in that year three firms put production machines of this type on the market—in England, Craven Brothers, of Manchester; in the United States, Gould & Eberhardt and Brown & Sharpe.

Brown & Sharpe in fact put out two types of machines in this same year, both using the formed milling-cutter method,[88] and both designed by Edward H. Parks. One of these (Fig. 55a) was for light small gears "for Sewing Machines

88. *Am. Mach.*, Aug. 6, 1914, p. 242.

and other light machinery," and was advertised[89] as a dual machine of two independent units mounted on one stand—one unit for spur gears and the other for bevel gears. In both units the gear blank revolved from tooth to tooth as each was cut, and the machine stopped automatically when the gear was completed. Brown & Sharpe records show that sixteen of these machines were made in 1879; the price was $650.

The larger machine was not shown until the catalogue of 1880.[90] It was designed for manufacture and repair "of Cotten and Woolen Machinery." As will be seen in Fig. 55b, it could cut either spur or bevel gears in sizes up to 18 inches in diameter, 3-inch face, and 8 diametral pitch. It, too, was fully automatic once the gear blank was in place and work set up.

FIG. 55A. BROWN & SHARPE AUTOMATIC-DUAL, 1877 (*Brown & Sharpe*)

FIG. 55B. BROWN & SHARPE AUTOMATIC—LARGE, 1877 (*Brown & Sharpe*)

89. Brown and Sharpe Catalogue, Jan. 1, 1879, p. 13.
90. Brown and Sharpe Catalogue, July 1, 1880, p. 1. *Am. Mach.*, Sept. 4, 1880, p. 1.

"The indexing is done by a worm-wheel and worm, and the movement of index for obtaining the number of teeth in wheel to be cut is performed by changeable ratchet wheels operated by a crank, adjusted to take a greater or less number of teeth, according to the pitch of wheel to be cut. The blank being put in place and the cutter head adjusted for length of stroke, the wheel is lowered by a screw, having a dial reading to thousandths of inches, until the proper depth of cut is obtained, when the cutter will pass through the blank and back through the space by a quick return movement; the blank is then moved for the next tooth, and so on until the whole wheel is cut. A number of blanks can be cut at one time when they are of uniform size. A steadying rest is provided for larger wheels, bearing against the rim. The cutter head is adjustable at any angle for cutting bevel wheels, the degrees being marked on a graduated arc, the motions being the same (without any change other than the raising of the head to an angle) as for spur gears. There is also provision for adjusting the cutter out of center. . . . A sufficient number of ratchet change wheels accompany the machine for dividing all numbers to one hundred, and all but the prime numbers over one hundred."

All this was available for $750, "Boxed and delivered at railroad or steamer, in Providence, without extra charge."

From 1877 onward the manufacturers of gear-cutting machines produced more and more the automatic types, except for those making very large gears.[91] The formed milling-cutter machine presented relatively simple problems to make it automatic, as did the templet and hobbing machines. It was the generating machine which required a great deal of ingenuity to make it a practical automatic machine. Bilgram's generating machine of 1884 did not appear in automatic form until the first years of this century.[92] But he did introduce a very ingenious gear train to give the advance required in indexing for various sizes of gears. In his automatic machines, instead of completing one tooth and then

91. See Flanders, *passim*.
92. Automatic for spur and spiral gears, *Am. Mach.*, Jan. 31, 1901, p. 110; automatic for bevel gears, *Am. Mach.*, Jan. 23, 1902, p. 114.

indexing to the next, the machine made a single cut on the first tooth, and then indexed successively all the way around making the same cut on each tooth. It then took the second cut required in generating the first tooth, and so on until the last cut had been taken on all the teeth, when it stopped to await another gear blank.

Because of the necessity of producing a machine capable of automatically cutting one or more types of gears, and gears of widely different sizes and pitches, these automatic machines were by the early 1900s very complex machines, requiring great skill in design, construction, and maintenance, yet capable of being operated in groups of several units by a relatively unskilled workman.[93] But they did make it possible to produce accurately cut gears of all commercial types cheaply and in quantity. Together with the Automatic Screw Machine, they were the forerunners of Automation,[94] and appeared out of conditions—economic, industrial, and technical—very much the same as have given rise to modern automation in our own day.

Precision—Gear Grinding, Gear Shaving, and Gear Measurement

The automobile introduced a demand, not only for new types of gears in quantity, but also for gears hardened to take the heavy wear resulting from driver usage and at the same time quiet in operation to satisfy public taste. It had already been discovered that it was not possible to cut a gear accurately in steel, harden it, and still have it retain the accuracy necessary for quiet running. Until the introduction of composition gears, the only solution was to harden a gear

93. See C. R. Gabriel's automatic spiral-gear cutter in his Patent No. 645,082 of Mar. 13, 1900, and O. J. Beale's automatic bevel-gear cutter in his Patent No. 795,021 of July 18, 1905, (app. Dec. 9, 1901).
94. See, for example, the automated gear hobber of Lees-Bradner based upon an electrically controlled in-out motion of the hob. Patent No. 2,364,932 of Dec. 12, 1944.

cut slightly oversize and then grind it to the desired dimensions. The technique of grinding metals had by this time already developed to the point where grinding was a successful commercial process, capable of giving high precision and excellent finish. But its adaptation to gear cutting, or more strictly, gear finishing, required a solution of one very important problem. In using a metallic cutting tool there is a certain amount of wear on the tool as it cuts, but this is small and leads, until automation, only to a problem of resharpening. However, in the grinding process the cutting tool—the grinding wheel—itself wears away quite rapidly. In fact, for good grinding results it is essential that it do so, at least to a controlled degree. This meant that for gear cutting the use of a formed grinding cutter, and even of a generating grinding wheel, required some means, not only of maintaining the proper form of the grinding wheel, but of compensating within the machine for the wear of the wheel.

The Leland & Faulconer Company were the pioneers in the grinding of gear teeth. They brought out first a machine to grind the teeth of a hardened bevel gear.[95] This was a templet machine. It was intended to produce hardened gears for chainless bicycles. The craze was for lightness, and the resulting gears gave equal strength with less weight for the "scorcher" of the day. Later this firm developed a generating machine, also for finishing hardened bevel gears. This used an emery wheel whose outer cross-section was that of an involute rack tooth. The principle is shown in Fig. 11f. The grinding wheel as it rotates was given a reciprocating motion across the face of the blank. The blank itself was given the rest of the necessary generating motions by being mounted on a swinging carrier, and also by being rotated on its own axis. The machine finished only one side of each tooth and then had to be reset to generate the other side.

Experiments intended to produce ground hardened gears for automobiles were carried out first in England by

95. *Am. Mach.*, June 29, 1899, p. 589.

Thomas Humpage.[96] He used a corundum worm in a hobbing machine to finish the hardened gears. The results were very good; and the wear on the corundum worm was, he tells us, very small. Unfortunately he does not tell us either the method or the relative cost of making the corundum worm. At any rate, his method was not widely adopted.

The crucial invention for the grinding of gears proved to be that of Ward and Taylor, of the Gear Grinding Machine Company, of Detroit. They provided a device by which the wear on the formed grinding wheel was corrected by two diamond points to maintain the correct form *automatically;* and the resulting wear on the grinding wheel was compensated for in the gear-grinding machine, also *automatically.*[97]

By 1909 all the basic types of gear-cutting machines had been built to use the grinding method, although Humpage's was the only one of the hobbing type. The formed-tool grinding machine of Upton & Gilman of Lowell, Massachusetts, was in use for truing up the teeth of cast spur gears. It did not, however, have any automatic means of maintaining the form of the grinding wheel.[98]

The Fellows Gear Shaper required, of course, hardened and ground cutters, made on his Gear-Cutter Machine.[99] This machine (Fig. 40b) uses the molding-generating principle in which an emery wheel with a plane face is kept straight by a diamond-point truing device which is an integral part of the machine. The details of the operation of this machine have been described above. It was not, however, used for grinding gears, only for making the cutters for the Fellows Gear Shaper.

The first machine for grinding the hardened gears for automobiles was the work of J. E. Reinecker of Germany.

96. Thomas Humpage, *Proc Inst. Mech. Eng.*, July 1908, pp. 674, 691, with comment by English engineers. The Swiss firm of Reishaur has recently put Humpage's method into a practical gear grinder.
97. See their Patent No. 888,675 of May 26, 1908.
98. Flanders, p. 65.
99. *Ibid.*, p. 99.

(Fig. 56). It used an emery wheel formed to the shape of the rack tooth. Unlike the Fellows machine, it used both sides of the wheel. Means were provided by which this rack-tooth form could be renewed as necessary, by diamond points, to any desired angle. The emery wheel itself moved vertically to grind the whole face of each tooth of the gear. At the same time the gear blank, carried on a vertical spindle, moved on a slide across the face of the wheel, while the blank also rotated. These movements were coordinated to give the effect of rolling the gear on an imaginary rack. One complete tooth space was done at a pass, the work then indexed, and another tooth was done on the return stroke of the slide. This cycle continued until the gear was completed. Gears 11¾ inches in diameter and with a 2-inch face could be done on the machine shown.

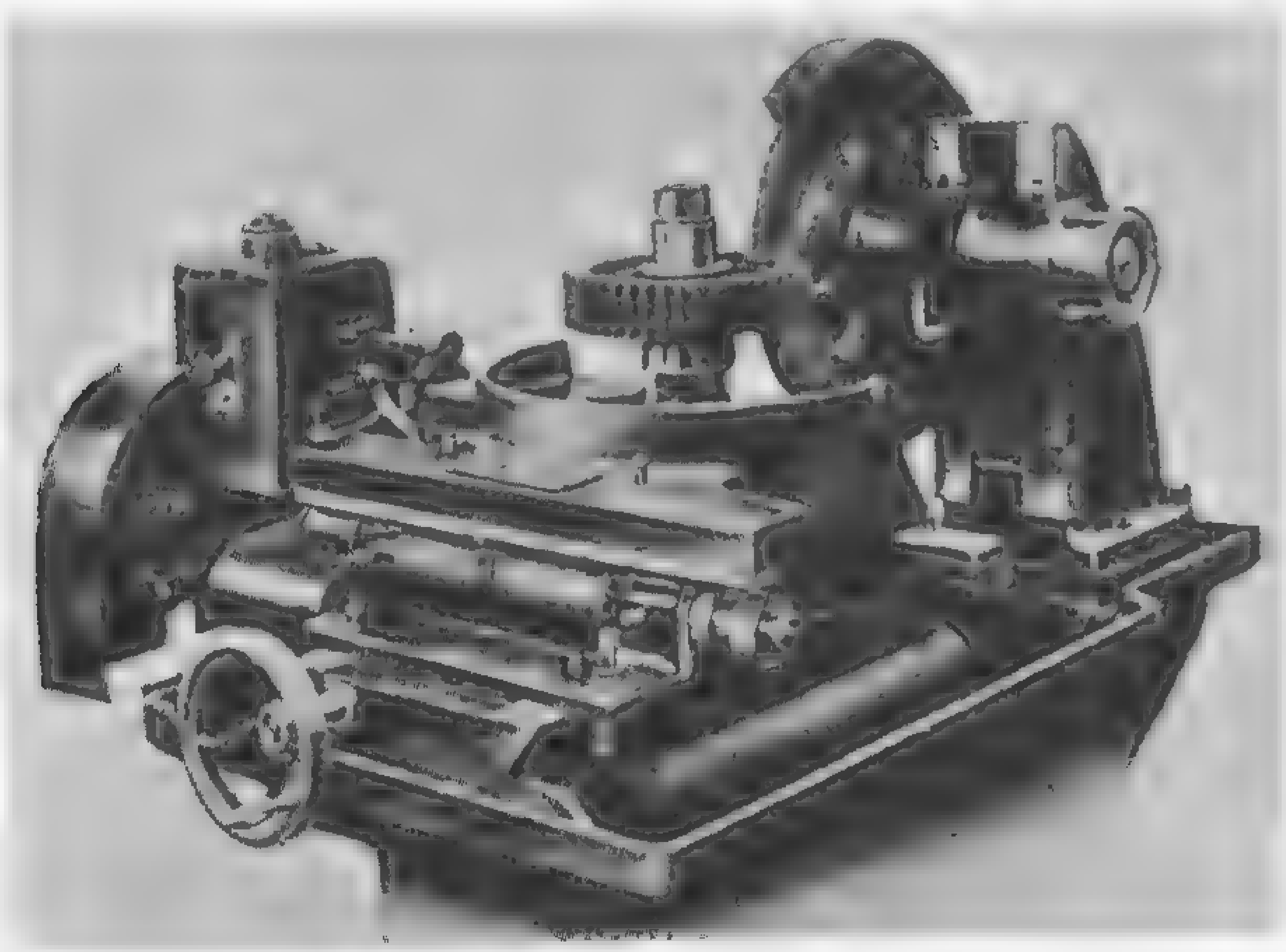


FIG. 56. REINECKER GEAR GRINDER (*Flanders*)

At about this same time the Oerlikon firm of Switzerland brought out a templet machine for grinding gears.[100] It was a modification of their regular templet planing machine, but used grinding wheels of a special composition such that a fairly large gear could be finished without appreciable change in the tooth form. The Oerlikon machine was widely used in automobile work.

100. Flanders, p. 266.

Even greater precision and better surface finish of hardened gears have been obtained in recent years by honing, using abrasive-impregnated plastic gear-shaped hones designed by the National Broach and Machine Co.[101] In present practice many gears are roughed out by hobbing, hardened, and then finished by gear grinding. This method is an extension of the important principle of production grinding first introduced for cylindrical grinding by C. H. Norton about 1900.[102]

Since the First World War there has been a demand for more precisely cut gears which are not hardened. This has resulted in a specialized gear-finishing machine known as a gear shaver. Its function is to take a very fine finishing cut over the teeth of gears already more rapidly and economically rough cut on conventional gear-cutting machines. A representative example of this machine is that of Pratt & Whitney.[103] The only new principle involved is the use of a wide cutting tool to do the shaving and to produce a tooth surface precise in its contours and having a very fine finish. The gear shaver therefore produces unhardened gears of the high accuracy required for high speed and noiseless operation.

In all this desire to obtain precision gears it became, of course, necessary to find means of accurate measurement of the product. The first devices were specialized micrometers which measured only the dimensions at the pitch circle. Various standardized wires were, and still are used with conventional micrometers for checking gear teeth. However, these devices measured at best only certain points on the tooth form.[104] It was not until the appearance of the Hartness

101. *The Tool Engineer,* April 1958, p. 195.

102. This important development will be included in a later monograph on the History of the Grinding Machine.

103. *Trans.* A.S.M.E., Oct. 12, 1928, MSP 50-17; and *Am. Mach.*, May 17, 1928, p. 810.

104. See B. F. Waterman, *Inspection of Gearing,* unpublished typescript of a talk given at the meeting of the American Gear Manufacturers Assn., Sept. 14-15, 1917, in Brown and Sharpe files.

Screw Thread Comparator in 1921,[105] and the adaptation of its principle to the gear problem that a genuine method of really checking gears was available.[106]

Conclusion

Today gears of an incredible variety of sizes and shapes enter into practically every machine that we use. Gears have been built to operate at tremendous speeds and loads. They appear in almost every device we use; if not in the device, they formed an important element in the machine which made it. All these gears, so central in the technology of our industrial society, had to be made on the gear-cutting machine—a tool that embodies at once fine mathematics, mechanical ingenuity, and machine work of the highest order.

105. See his Patent No. 1,377,068 of May 3, 1921.
106. This story will be given in detail in a later monograph on the History of Shop Precision of Measurement.

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